OPTI 435/535 Homework 1 Solutions

1.

Snellen Acuity (feet)	Snellen Acuity (metric)	LogMAR Acuity
20/10	6/3	-0.3
20/12.5	6/4	-0.2
20/16	6/5	-0.1
20/20	6/6	0
20/25	6/8	0.1
20/32	6/10	0.2
20/40	6/12	0.3
20/50	6/15	0.4
20/63	6/19	0.5
20/80	6/24	0.6
20/100	6/30	0.7
20/125	6/38	0.8
20/160	6/48	0.9
20/200	6/60	1.0
20/400	6/120	1.3

The key here is that each step of 01 in logMAR is a just noticeable difference in visual performance. A visual acuity of 20/400 is legally blind.

2. Find the positions of the Cardinal Points (F, F', P, P', N, N') and the positions of the entrance and exit pupils (E, E') for the Gullstrand **Accommodated** Schematic Eye.

If we enter the accommodated eye model values into Zemax then

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CARDINAL POINTS:

Object space positions are measured with respect to surface 1.
Image space positions are measured with respect to the image surf
The index in both the object space and image space is considered.

                           Object Space        Image Space
W = 0.587562 (Primary)
Focal Length          :       -14.775094          14.775094
Focal Planes          :       -12.953963          -1.694502
Principal Planes      :         1.821130         -16.469596
Anti-Principal Planes :       -27.729057          13.080591
Nodal Planes          :         1.821130         -16.469596
Anti-Nodal Planes     :       -27.729057          13.080591


Entrance Pupil Diameter :              3
Entrance Pupil Position :       2.660778
Exit Pupil Diameter     :       2.838682
Exit Pupil Position     :       -15.6751
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There's likely to be some difference between this and other programs that can be related to the model of the index of refraction. Here we have chosen the d wavelength so no dispersion should exist from the MIL numbers used to give index of refraction.

3. Based on the results of question 2, where is the near point of the eye? Also, what is the power of the accommodated crystalline lens?

Again, in Zemax if we shorten the object distance to 130.76 mm, then this point is conjugate to the retina. This point is therefore the near point of the Gullstrand eye since it is fully accommodated. The power of the crystalline lens is calculated as follows:

$$\varphi_1 = \frac{n_1' - n_1}{R_1} = \frac{1.427 - 1.337}{6} = 15D$$

$$\varphi_2 = \frac{n_2' - n_2}{R_2} = \frac{1.336 - 1.427}{-5.5} = 16.55D$$

$$\varphi = \varphi_1 + \varphi_2 - \frac{t_1}{n_1'}\varphi_1\varphi_2 = 30.76D$$

********************************* Grads Only **********************************

4. The Point Spread Function (PSF) of the eye is the squared-modulus of the Fourier Transform of the Pupil Function. In a rotationally symmetric, aberration-free system, the PSF is given by

$$\mathrm{PSF} = \left| 2\pi \int_0^{r_{max}} P(r) J_0(2\pi\rho r) r\,dr \right|^2$$

where r_{max} is the radius of the pupil and P(r) is the transmission function of the pupil.

(a) What is the PSF for a uniform transmission function (i.e. P(r)=1) and r_{max} = 4 mm?

In this case, the PSF is the square of the Hankel transform of a cylinder function

$$\mathrm{PSF} = \left| H\left\{ \mathrm{cyl}\left(\frac{r}{2r_{max}} \right) \right\} \right|^2 = \left| \pi r_{max}^2 \,\mathrm{somb}(2r_{max}\rho) \right|^2$$

(b) What is the PSF for a Stiles-Crawford apodized pupil with $P(r) = \exp(-0.105r^2)$. Assume r_{max} very large for this part.

If we let r_{max} approach infinity, then the PSF is the square of the Hankel transform of a gaussian function

$$\text{PSF} = \left| H\left\{ \text{gaus}\left(\frac{r}{\sqrt{\frac{\pi}{0.105}}} \right) \right\} \right|^2 = \left| \frac{\pi}{0.105}\, \text{gaus}\left(\frac{\rho}{\sqrt{\frac{0.105}{\pi}}} \right) \right|^2$$

(c) Plot the results and compare the widths of the PSFs at half max.

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In[25]:= f1 = (16*π)^2*(BesselJ[1, 8*π*ρ] / (8*π*ρ))
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$$\text{Out[25]=}\quad \frac{4\,\text{BesselJ}[1, 8\pi\rho]^2}{\rho^2}$$

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In[28]:= Plot[f1, {ρ, -.5, .5}, PlotRange → {0, 650}]
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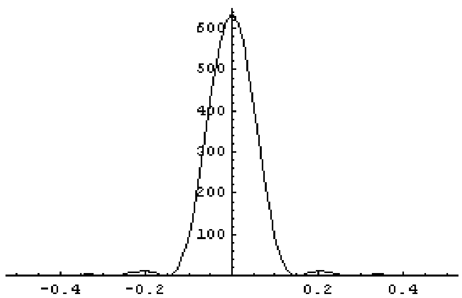


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In[29]:= f2 = ((π / 0.105)*Exp[-(π^2 / 0.105)*ρ^2])^2
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$$\text{Out[29]=}\quad 895.202\, e^{-187.992\,\rho^2}$$

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In[32]:= Plot[f2, {ρ, -.5, .5}, PlotRange → {0, 900}]
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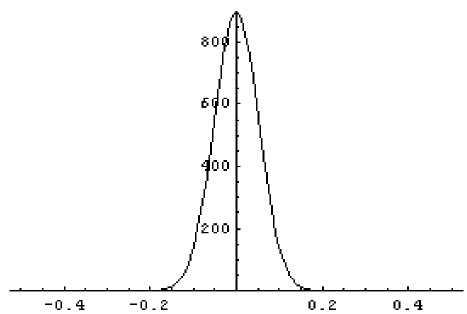

The key here is that the Stiles-Crawford effect gives about the same diameter spot, but eliminates the rings associated with the Airy disk pattern.