OPTI 435/535 Homwork 2 Solutions 2008

■ Problem 1

The wavefront error is given by

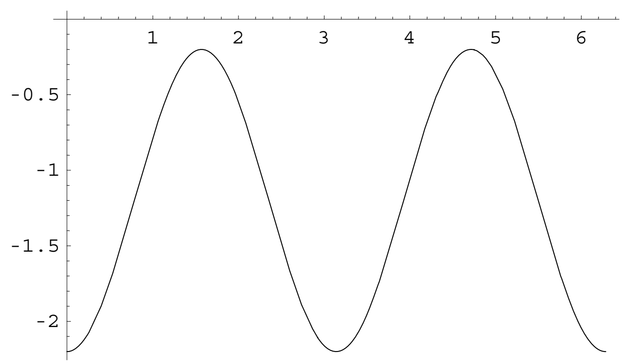
```
In[2]:= W = -0.001*r^2 - 0.0005*r^2*Cos[2*θ] + 0.00005*r^4
```

Out[2]= $-0.001\, r^2 + 0.00005\, r^4 - 0.0005\, r^2 \operatorname{Cos}[2\,\theta]$

```
In[4]:= dϕ[r_, θ_] = Simplify[(1000/r)*D[W, r]]
```

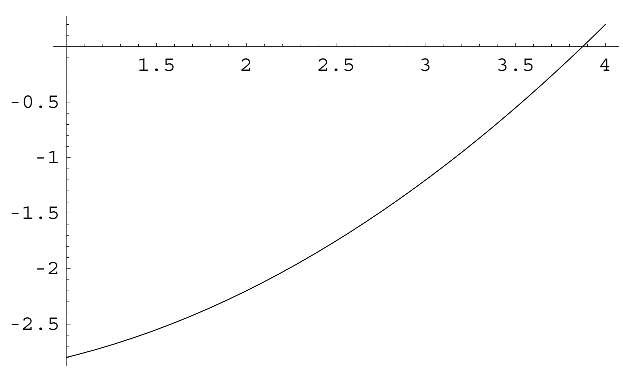
Out[4]= $-2. + 0.2\, r^2 - 1.\, \operatorname{Cos}[2\,\theta]$

```
In[5]:= Plot[dϕ[2, θ], {θ, 0, 2*π}]
```



Out[5]= - Graphics -

```
In[6]:= Plot[dϕ[r, 0], {r, 1, 4}]
```



Out[6]= - Graphics -

■ Problem 2

The sag of a paraboloid is given by

In[7]:= **z = (x^2 + y^2) / (2*R)**

Out[7]= $\frac{x^2+y^2}{2\,R}$

First, calculate the first and second derivatives

In[58]:=
```
dzdx = D[z, x]
dzdy = D[z, y]
d2zdx2 = D[dzdx, x]
d2zdy2 = D[dzdy, y]
d2zdxdy = D[dzdx, y]
```

Out[58]= $\frac{x}{R}$

Out[59]= $\frac{y}{R}$

Out[60]= $\frac{1}{R}$

Out[61]= $\frac{1}{R}$

Out[62]= 0

Next, calculate the First and Second Fundamental Forms

In[63]:=
```
E0 = 1 + dzdx^2
F0 = dzdx*dzdy
G0 = 1 + dzdy^2
L0 = d2zdx2 / Sqrt[E0*G0 - F0^2]
M0 = d2zdxdy / Sqrt[E0*G0 - F0^2]
N0 = d2zdy2 / Sqrt[E0*G0 - F0^2]
```

Out[63]= $1+\frac{x^2}{R^2}$

Out[64]= $\frac{x\,y}{R^2}$

Out[65]= $1+\frac{y^2}{R^2}$

Out[66]= $\frac{1}{R\sqrt{-\frac{x^2\,y^2}{R^4}+\left(1+\frac{x^2}{R^2}\right)\left(1+\frac{y^2}{R^2}\right)}}$

Out[67]= 0

Out[68]= $\frac{1}{R\sqrt{-\frac{x^2\,y^2}{R^4}+\left(1+\frac{x^2}{R^2}\right)\left(1+\frac{y^2}{R^2}\right)}}$

Calculate the Mean Curvature

In[70]:= `H0 = Simplify[(E0 * N0 + G0 * L0 + 2 * F0 * M0) / (2 * (E0 * G0 - F0 ^ 2))]`

Out[70]= $$\frac{2 R^2 + x^2 + y^2}{2 R^3 \left(\frac{R^2+x^2+y^2}{R^2}\right)^{3/2}}$$

In[71]:= $$H0 = \frac{2 R^2 + x^2 + y^2}{2 (R^2 + x^2 + y^2)^{3/2}}$$

Out[71]= $$\frac{2 R^2 + x^2 + y^2}{2 (R^2 + x^2 + y^2)^{3/2}}$$

Finally, evaluate the Mean Curvature at the origin

In[75]:= `Simplify[H0 /. {x → 0, y → 0}, {R ∈ Reals, R > 0}]`

Out[75]= $$\frac{1}{R}$$

which is the same as the curvature of a sphere with radius R

■ Problem 3

The sag of a biconic surface is given by

In[76]:= `z = (x ^ 2 / Rx + y ^ 2 / Ry) / (1 + Sqrt[1 - (1 + Kx) * x ^ 2 / Rx ^ 2 - (1 + Ky) * y ^ 2 / Ry ^ 2])`

Out[76]= $$\frac{\frac{x^2}{Rx} + \frac{y^2}{Ry}}{1 + \sqrt{1 - \frac{(1+Kx)\, x^2}{Rx^2} - \frac{(1+Ky)\, y^2}{Ry^2}}}$$

First, convert to polar coordinates

In[77]:= `Simplify[z /. {x → r * Cos[θ], y → r * Sin[θ]}]`

Out[77]= $$\frac{r^2 \left(Ry\, Cos[\theta]^2 + Rx\, Sin[\theta]^2\right)}{Rx\, Ry \left(1 + \sqrt{1 - \frac{(1+Kx)\, r^2\, Cos[\theta]^2}{Rx^2} - \frac{(1+Ky)\, r^2\, Sin[\theta]^2}{Ry^2}}\right)}$$

Examining the numerator,

it is clear that $R_{\text{eff}} = \frac{R_x R_y}{R_y \cos^2(\theta) + R_x \sin^2(\theta)}$. Furthermore,

factor out the r^2 under the radical in the denominator

In[78]:= $$z = \frac{r^2 / Reff}{\left(1 + \sqrt{1 - Collect\left[\left(\frac{(1+Kx)\, r^2\, Cos[\theta]^2}{Rx^2} + \frac{(1+Ky)\, r^2\, Sin[\theta]^2}{Ry^2}\right), r\right]}\right)}$$

Out[78]= $$\frac{r^2}{Reff \left(1 + \sqrt{1 - r^2 \left(\frac{(1+Kx)\, Cos[\theta]^2}{Rx^2} + \frac{(1+Ky)\, Sin[\theta]^2}{Ry^2}\right)}\right)}$$

The quantity in parentheses under the radical has to equal $\frac{K_{eff}+1}{R_{eff}^2}$

In[85]:= `FullSimplify[Solve[(Keff + 1) / Reff ^ 2 == (1 + Kx) Cos[θ]^2/Rx^2 + (1 + Ky) Sin[θ]^2/Ry^2, Keff]]`

Out[85]= $\left\{\left\{\text{Keff} \to \text{Reff}^2 \left(-\frac{1}{\text{Reff}^2} + \frac{(1+\text{Kx}) \text{Cos}[\theta]^2}{\text{Rx}^2} + \frac{(1+\text{Ky}) \text{Sin}[\theta]^2}{\text{Ry}^2}\right)\right\}\right\}$

In[88]:= `Keff = Simplify[Distribute[((Rx^2 * Ry^2) / (Ry * Cos[θ]^2 + Rx * Sin[θ]^2)^2) * (-1/((Rx^2 * Ry^2) / (Ry * Cos[θ]^2 + Rx * Sin[θ]^2)^2) + (1 + Kx) Cos[θ]^2/Rx^2 + (1 + Ky) Sin[θ]^2/Ry^2)]]`

Out[88]= $-1 + \frac{(1+\text{Kx}) \text{Ry}^2 \text{Cos}[\theta]^2}{(\text{Ry Cos}[\theta]^2 + \text{Rx Sin}[\theta]^2)^2} + \frac{(1+\text{Ky}) \text{Rx}^2 \text{Sin}[\theta]^2}{(\text{Ry Cos}[\theta]^2 + \text{Rx Sin}[\theta]^2)^2}$

In[89]:= `-1 + Together[(1 + Kx) Ry^2 Cos[θ]^2/(Ry Cos[θ]^2 + Rx Sin[θ]^2)^2 + (1 + Ky) Rx^2 Sin[θ]^2/(Ry Cos[θ]^2 + Rx Sin[θ]^2)^2]`

Out[89]= $-1 + \frac{\text{Ry}^2 \text{Cos}[\theta]^2 + \text{Kx Ry}^2 \text{Cos}[\theta]^2 + \text{Rx}^2 \text{Sin}[\theta]^2 + \text{Ky Rx}^2 \text{Sin}[\theta]^2}{(\text{Ry Cos}[\theta]^2 + \text{Rx Sin}[\theta]^2)^2}$

In[91]:= `-1 + Collect[Ry^2 Cos[θ]^2 + Kx Ry^2 Cos[θ]^2 + Rx^2 Sin[θ]^2 + Ky Rx^2 Sin[θ]^2, {Cos[θ], Sin[θ], Rx, Ry}] / (Ry Cos[θ]^2 + Rx Sin[θ]^2)^2`

Out[91]= $-1 + \frac{(1+\text{Kx}) \text{Ry}^2 \text{Cos}[\theta]^2 + (1+\text{Ky}) \text{Rx}^2 \text{Sin}[\theta]^2}{(\text{Ry Cos}[\theta]^2 + \text{Rx Sin}[\theta]^2)^2}$

■ Problem 4

Below is a zpl macro that calcualtes the spherical aberration of an eye model in Zemax. Similar macros can be constructed in other raytracing program, or the procedure can be done manually.

```
nsurf=nsur()
n=indx(nsurf-1)
print "Pupil Diameter    LSA"
for i=1,8,1
        atyp=0
        aval=i
        getsystemdata 1
        pf=n*vec1(7)
        pv=pf-thic(nsurf-1)
        raytrace 0,0,0,1
        tanu=tang(acos(rayn(nsurf-1)))
        y=rayy(nsurf-1)
        z=rayz(nsurf-1)
        vm=y/tanu+z
        pm=pv+vm
        lsa=1000*n*(1/pm-1/pf)
        print,i,"          ",lsa
next
```

We can dissect the lines of this macro as follows:

Get the refractive index of the vitreous, nsurf caluclates the total number of surfaces in the eye model and indx retrieves the refractive index

```
nsurf=nsur()
n=indx(nsurf-1)
```

Loop through the various pupil diameters from 1 mm to 8 mm in steps of 1mm. This is equivalent to pupil radii from 0.5 mm to 4 mm.

```
for i=1,8,1
next
```

Set the aperture type to Entrance Pupil Diameter

```
atyp=0
```

Set the diameter of the Entrance Pupil to the loop variable i

```
aval=i
```

Get a copy of the System Data. This places a bunch of useful data regarding the system into an array called

vec1
getsystemdata 1

The 7th item in the array is the focal length. Remember to scale by the index of the vitreous to get P'F'
pf=n*vec1(7)

Calculate the position of the rear principal plane relative to the posterior crytalline lens
pv=pf-thic(nsurf-1)

Raytrace a marginal ray. Its height is indirectly set by the current Entrance Pupil diameter
raytrace 0,0,0,1

Calculate the marginal ray angle and intersection coordinates at the posterior crystalline lens
tanu=tang(acos(rayn(nsurf-1)))
y=rayy(nsurf-1)
z=rayz(nsurf-1)

Calculate the position of the marginal focus relative to the posterior crystalline lens
vm=y/tanu+z

Calculate the position of the marginal focus relative to the rear principal plane. This is P'M'
pm=pv+vm

Finally, calculate and print the longitudinal spherical aberration
lsa=1000*n*(1/pm-1/pf)
print,i,"   ",lsa

For the Gullstrand-LeGrand Model, this macro gives the following result

In[105]:=
```
LSA1 = {{1.0000, 0.0904},
  {2.0000, 0.3656},
  {3.0000, 0.8386},
  {4.0000, 1.5336},
  {5.0000, 2.4911},
  {6.0000, 3.7780},
  {7.0000, 5.5100},
  {8.0000, 7.9068}}
```

Out[105]=
```
{{1., 0.0904}, {2., 0.3656}, {3., 0.8386},
 {4., 1.5336}, {5., 2.4911}, {6., 3.778}, {7., 5.51}, {8., 7.9068}}
```

For the Arizona Eye Model, the macro gives

Pupil Diameter LSA

1.0000 0.0275
2.0000 0.1111
3.0000 0.2620
4.0000 0.4965
5.0000 0.8363
6.0000 1.3083
7.0000 1.9463
8.0000 2.7940

In[106]:=

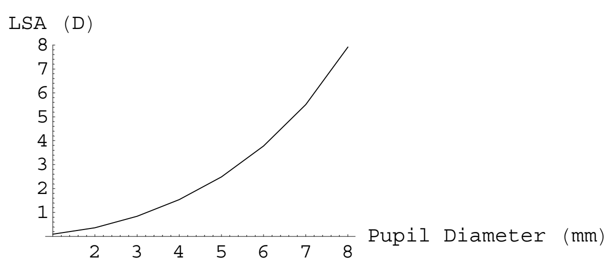
```
LSA2 = {{1.0000, 0.0275},
{2.0000, 0.1111},
{3.0000, 0.2620},
{4.0000, 0.4965},
{5.0000, 0.8363},
{6.0000, 1.3083},
{7.0000, 1.9463},
{8.0000, 2.7940}}
```

Out[106]=

```
{{1., 0.0275}, {2., 0.1111}, {3., 0.262}, {4., 0.4965},
 {5., 0.8363}, {6., 1.3083}, {7., 1.9463}, {8., 2.794}}
```

In[117]:=

```
ListPlot[LSA1, PlotJoined → True,
 AxesLabel → {"Pupil Diameter (mm)", "LSA (D)"}, PlotRange → {0, 8}]
```

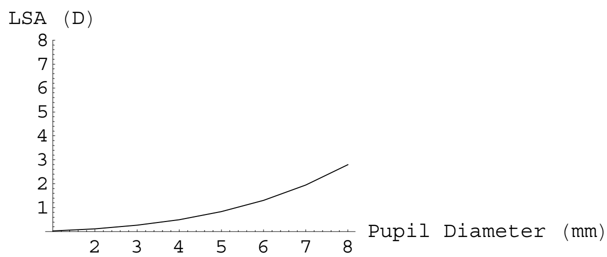


Out[117]=

```
- Graphics -
```

In[118]:=

```
ListPlot[LSA2, PlotJoined → True,
  AxesLabel → {"Pupil Diameter (mm)", "LSA (D)"}, PlotRange → {0, 8}]
```



Out[118]=

```
- Graphics -
```