Homework 3 Solutions

Questions 1

Suppose an eye has a wavefront error

$W(\rho, \theta) = 0.001\, Z_4^0\, (\rho, \theta)$

where ρ = r / r_{max} and r_{max} = 3 mm. If the pupil shrinks to r_{max} = 1.5 mm, what is the new wavefront error in terms of Zernike polynomials?

```
In[1]:= W[ρ_, θ_] = (1 / 1000) * Sqrt[5] * (6 ρ^4 - 6 ρ^2 + 1)
```

Out[1]= $\frac{1 - 6\rho^2 + 6\rho^4}{200\sqrt{5}}$

Convert to unnormalized coordinates

```
In[3]:= W[r / 3, θ]
```

Out[3]= $\frac{1 - \frac{2 r^2}{3} + \frac{2 r^4}{27}}{200\sqrt{5}}$

For the new pupil size ρ = r / 1.5, → r = 1.5ρ

```
In[4]:= W[r / 3, θ] /. r → 3 ρ / 2
```

Out[4]= $\frac{1 - \frac{3\rho^2}{2} + \frac{3\rho^4}{8}}{200\sqrt{5}}$

Now project this wavefront onto the various Zernike terms to get the coefficients. The coefficient for $Z_4^0$is

In[11]:= $2\int_0^1 \left(\frac{1 - \frac{3\rho^2}{2} + \frac{3\rho^4}{8}}{200\sqrt{5}}\right) * \mathrm{Sqrt}[5] * (6\rho\text{^}4 - 6\rho\text{^}2 + 1)\,\rho\, d\rho$

Out[11]= $\frac{1}{16\,000}$

The coefficient for $Z_2^0$is

In[12]:= $2\int_0^1 \left(\frac{1 - \frac{3\rho^2}{2} + \frac{3\rho^4}{8}}{200\sqrt{5}}\right) * \mathrm{Sqrt}[3] * (2\rho\text{^}2 - 1)\,\rho\, d\rho$

Out[12]= $-\frac{3\sqrt{\frac{3}{5}}}{3200}$

The coefficient for Z_0^0 is

In[13]:=
$$2\int_0^1 \left(\frac{1-\frac{3\rho^2}{2}+\frac{3\rho^4}{8}}{200\sqrt{5}}\right)\rho\, d\rho$$

Out[13]=
$$\frac{3}{1600\sqrt{5}}$$

Question 2

The power of the cornea is given by

```
ϕ1 = 1000 * (1.336 - 1) / 7.8
```

```
43.0769
```

For an object at infinity, the cornea (by itself) forms an image

```
Solve[1.336 / L1p == ϕ1, L1p]
```

```
{{L1p → 0.0310143}}
```

```
L1p = 0.031014285714285706`
```

```
0.0310143
```

We can then use the imaging equation do determine where the IOL sits

```
Solve[1.336 / (0.024 - d) - 1.336 / (L1p - d) == 20.0, d]
```

```
{{d → 0.00557875}, {d → 0.0494355}}
```

Choose the first solution since it is in the eye. So the IOL sits 5.58 mm behind the cornea.

For the object at 33cm, the cornea (by itself) forms an image at

```
Clear[L1p]
```

```
Solve[1.336 / L1p + 1 / .33 == ϕ1, L1p]
```

```
{{L1p → 0.0333611}}
```

```
L1p = 0.033361117578579735`
```

```
0.0333611
```

Calculate the IOL position d for the new object distance

```
Solve[1.336 / (0.024 - d) - 1.336 / (L1p - d) == 20.0, d]
```

```
{{d → 0.00323984}, {d → 0.0541213}}
```

The IOL now sits 3.24 mm behind the cornea, so it had to move 2.34 mm

Question 3

PMMA has an index of n = 1.49. The thickness t = 1 mm. Equiconvex means that the front and back radii R1 = -R2. The power of each surface is given by

```
n = 1.49
ϕ1 = (n - 1.336) / R1
ϕ2 = -(1.336 - n) / R1
```

```
1.49
```

$$\frac{0.154}{R1}$$

$$\frac{0.154}{R1}$$

Next solve the thick lens power equation for R1

```
t = 0.001
ϕ = 20
Solve[ϕ == ϕ1 + ϕ2 - t * ϕ1 * ϕ2 / n, R1]
```

```
0.001

20

{{R1 → 0.0000518524}, {R1 → 0.0153481}}
```

So R1=15.35 mm. In air, the surface powers would be

```
ϕ1 = (n - 1) / 0.015348147558319293`
ϕ2 = -(1 - n) / 0.015348147558319293`
```

```
31.9257

31.9257
```

and the total power (D) would be

```
ϕ = ϕ1 + ϕ2 - t * ϕ1 * ϕ2 / n
```

```
63.1673
```

Question 4