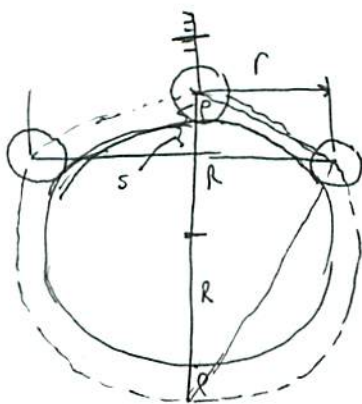


3. NON-INTERFEROMETRIC TESTING

3.1 SURFACE RADIUS OF CURVATURE

3.1.1 ~~SURFACE RADIUS~~ GENEVA GAUGE (SHOW SLIDE)

A Geneva gauge is a simple device for measuring the radius of curvature of a surface. Suppose we have three legs that are tipped with spheres of radius p . The outer two legs are fixed and separated by $2r$.



The center leg moves so that when the device is placed on a spherical surface, we can measure the displacement s of the center leg relative to the outer legs. From the drawing, using similar triangles

$$\frac{r}{2(R+p)-s} = \frac{s}{r} \Rightarrow r^2 = 2s(R+p) - s^2$$

$$R = \frac{r^2 + s^2}{2s} - p \quad \text{FOR CONVEX SURFACE}$$

$$R = \frac{r^2 + s^2}{2s} + p \quad \text{FOR CONCAVE SURFACE}$$

A Geneva gauge uses this principle but typically gives "power" under the assumption of a "crown" glass with $n = 1.523$

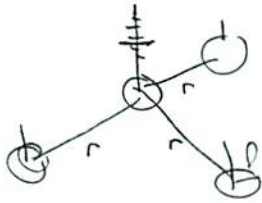
$$\text{Power } \phi_g = \frac{0.523}{R}$$

If we have a different material of known index n , then the true power is

$$\phi = \left(\frac{n-1}{0.523} \right) \phi_g$$

If gauge is not \perp to surface, the wrong answer is obtained

3.1.2 Spherometer is a similar device to the Geneva gauge, but typically has 3 outer legs in a triangular pattern to provide stability. It avoids errors related to tilting, but may miss astigmatism.



(SHOW SLIDE)

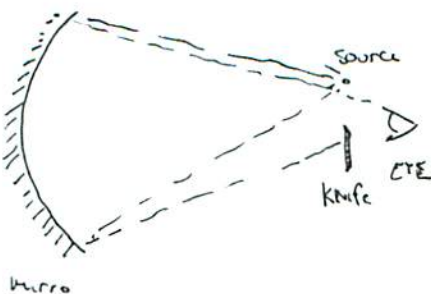
3.2 Wavefronts

Non-interferometric testing of wavefronts are usually fast, inexpensive and qualitative means for assessing the aberrations of a system. There are two key concepts to understanding how these tests work.

- ① A mask is placed near focus (but not necessarily at focus). The mask has a known pattern which blocks certain rays. We need to know the position of rays in the mask plane to determine if they are blocked or passed \rightarrow transverse ray error $\epsilon_x(p_x, p_y)$ $\epsilon_y(p_x, p_y)$
- ② For the Foucault, wire and Ronchi test, we view the exit pupil of the system so we see dark regions at points (p_x, p_y) for rays that are blocked and light regions at points (p_x, p_y) for rays that pass. For the Hartmann test, the pattern is recorded in the mask plane.

3.2.1 Foucault Knife Edge Test

A knife edge (e.g. razor blade) is used to block a portion of the beam near focus. The eye looks back into the system at the exit pupil and observes the pattern.



We are concerned with fabrication errors which destroy the rotational symmetry of the system, so we'll look at

w_{30} , w_{31} and w_{32}

We need to know the transverse ray error in the plane of the knife

$$\xi_x = \frac{-R}{r_{max}} \left[2W_{20} \rho_x + 4W_{40} (\rho_x^3 + \rho_x \rho_y^2) + 2W_{31} \rho_x \rho_y \right]$$

$$\xi_y = \frac{-R}{r_{max}} \left[2W_{02} \rho_y + 4W_{40} (\rho_x^2 \rho_y + \rho_y^3) + W_{31} (\rho_x^2 + 3\rho_y^2) + 2W_{22} \rho_y \right]$$

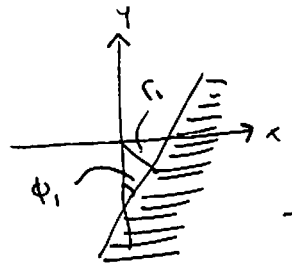
Need W_{20} term to handle defocus of knife from paraxial image plane

From page (52) of the notes we also showed

$$\delta z = 8 (f/\#)^2 W_{20}$$

where δz is the distance between the mask and the paraxial image plane.

Next, we need a convenient way to describe the knife edge



r_1 = distance of knife edge from optical axis

ϕ_1 = orientation of knife edge

The boundary of the knife is given by

$$x \cos \phi_1 - y \sin \phi_1 = r_1$$

where x and y are coordinates in the mask plane.

The transmission of the mask when W_{40} , W_{31} and W_{22} are present is given by

$$T = \begin{cases} 1 & \text{for } \xi_x \cos \phi_1 - \xi_y \sin \phi_1 < r_1 \\ 0 & \text{for } \xi_x \cos \phi_1 - \xi_y \sin \phi_1 \geq r_1 \end{cases}$$

$$T(\rho_x, \rho_y) = \begin{cases} 1 & \text{for } \frac{-R}{r_{max}} \cos \phi_1 \left[\frac{\delta z}{4(f/\#)^2} \rho_x + 4W_{40} (\rho_x^3 + \rho_x \rho_y^2) + 2W_{31} \rho_x \rho_y \right] \\ & - \frac{R}{r_{max}} \sin \phi_1 \left[\frac{\delta z}{4(f/\#)^2} \rho_y + 4W_{40} (\rho_x^2 \rho_y + \rho_y^3) + W_{31} (\rho_x^2 + 3\rho_y^2) + 2W_{22} \rho_y \right] < r_1 \\ 0 & \text{otherwise} \end{cases}$$

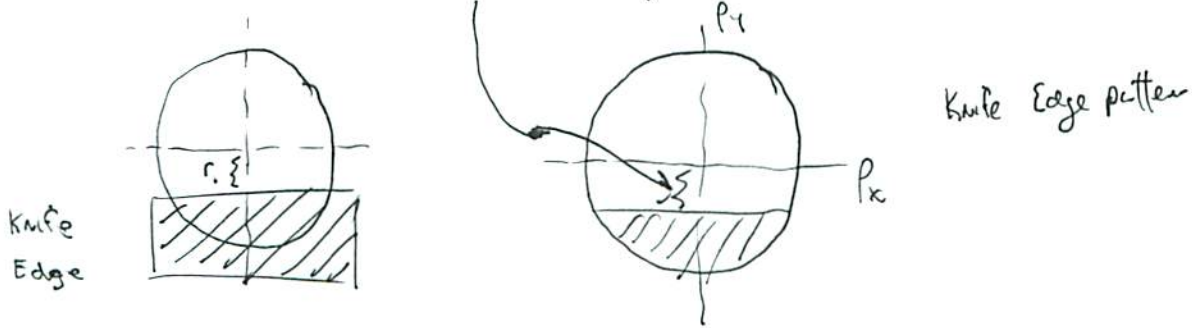
Can plot this for $\rho_x^2 + \rho_y^2 \leq 1$ i.e. over the exit pupil

Example (1) $\phi_1 = \frac{\pi}{2}$, Astigmatism Only

Boundary of shadow pattern occurs when

$$\frac{-R}{r_{max}} \left[\frac{\delta z}{4(f/\#)^2} p_y + 2w_{22} p_y \right] = r_1$$

$$p_y = \frac{r_1}{\frac{-R}{r_{max}} \left[\frac{\delta z}{4(f/\#)^2} + 2w_{22} \right]} \Rightarrow \text{Horizontal line}$$



Example (2) $\phi_1 = \frac{\pi}{2}$, Coma only

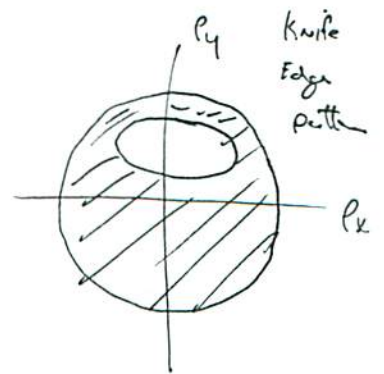
Boundary occurs when

$$\frac{-R}{r_{max}} \left[\frac{\delta z}{4(f/\#)^2} p_y + w_{31} (p_x^2 + 3p_y^2) \right] = r_1$$

$$p_y^2 + \frac{\delta z}{12(f/\#)^2 w_{31}} p_y + \frac{p_x^2}{3} = -\frac{r_1 r_{max}}{3R w_{31}}$$

Define $A = \frac{\delta z}{24(f/\#)^2 w_{31}}$; $B = \frac{-r_1 r_{max}}{3R w_{31}}$

$$p_y^2 + 2A p_y + A^2 + \frac{p_x^2}{3} = B + A^2$$



$$\frac{(p_y + A)^2}{B + A^2} + \frac{p_x^2}{3(B + A^2)} = 1$$

Equation of ellipse centered at $-A$
 semi-major axis = $\sqrt{3(B + A^2)}$
 semi-minor axis = $\sqrt{B + A^2}$

Example ③ $\phi_1 = 0$, coma only

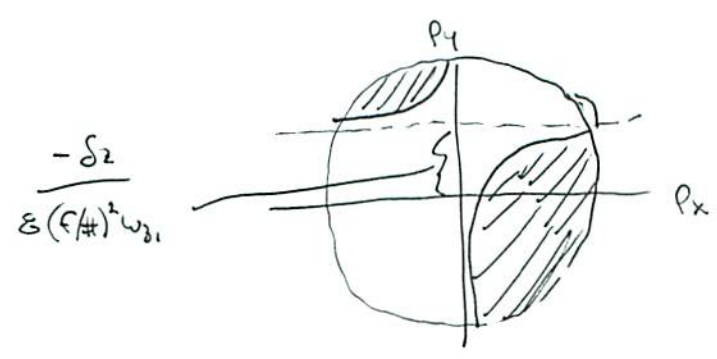
boundary occurs where

$$\frac{-R}{r_{max}} \left[\frac{\delta z}{4(f/\#)^2} p_x + 2\omega_3 p_x p_y \right] = r_1$$

$$p_x p_y + \frac{\delta z}{8(f/\#)^2 \omega_3} p_x = - \frac{r_1 r_{max}}{2R \omega_3}$$

$$p_x \left(p_y + \frac{\delta z}{8(f/\#)^2 \omega_3} \right) = - \frac{r_1 r_{max}}{2R \omega_3}$$

Eq. of hyperbola with horizontal and vertical asymptotes

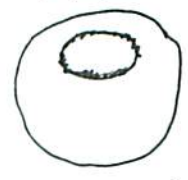


3.2.2 Wire Test - Same concept as knife edge test, but thin wire used instead of knife. The patterns are similar, but only the boundaries between the light and dark regions are seen. From the previous examples, the

Example 1



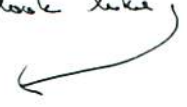
Example 2



Example 3



wire test patterns would look like



3.2.3 Ronchi Test - Uses Ronchi Ruling as mask. This is like wire test with multiple wires simultaneously, which is then is like knife edge test with multiple values of r_1 and only the boundaries seen,



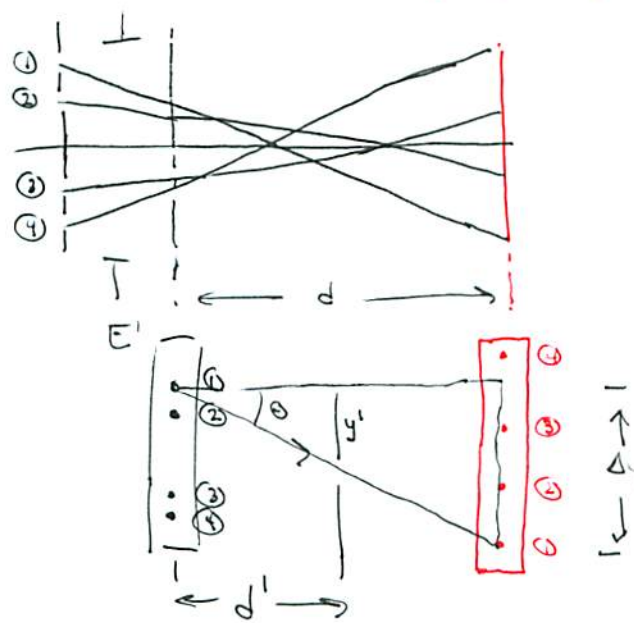
mask

SHOW SLIDES
SHOW MOVIE

3.2.4 HARTMANN SCREEN TEST - This test somewhat inverts the masking procedure seen in the previous tests. Here the mask is placed in the pupil to form discrete values of p_x and p_y and then $\xi_x(p_x, p_y)$, $\xi_y(p_x, p_y)$ measured at planes inside and outside of focus.

Effectively, this is a spot diagram. If we capture patterns outside of the caustic, the spots can be related back to their pupil coordinates p_x, p_y . Two or more planes outside of the caustic define the ray ~~geometry~~ trajectory.

SHOW SLIDES



The trajectory of ray 1 is defined by d and the change in ray height Δy between the two planes.

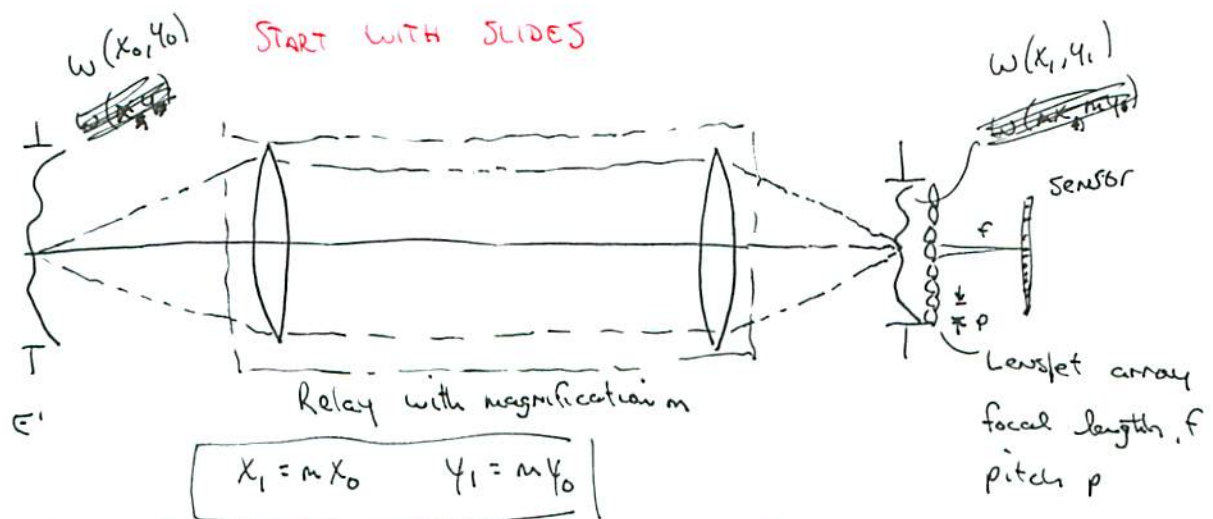
$$\tan \theta = \frac{\Delta y}{d}$$

Let's you determine the ray height for any position in space

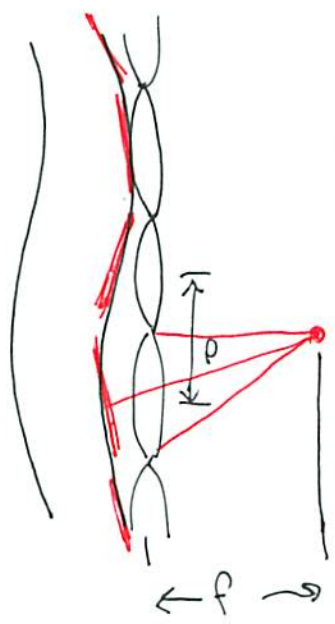
$$y' = \frac{d'}{d} \Delta y$$

In regions where the rays cross (caustic), relating which spot belongs to which hole in the mask becomes challenging. The general technique for this test is to measure two planes outside of the caustic and the predict how the rays cross the optical axis. Good systems have the rays cross at the same location.

3.2.5 SHACK-HARTMANN SENSOR

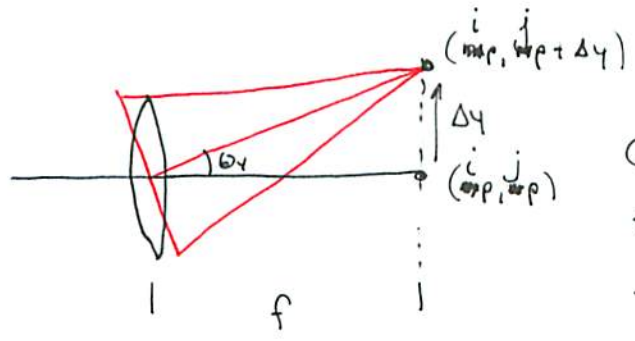


Wavefront from exit pupil is relayed to lenslet array. If relay has magnification m , then a wavefront error $w(x_0, y_0)$ gets projected onto the lenslet array as $w(x_1, y_1)$. This means the wavefront is compressed in the transverse direction, but the phase remains the same.



Assume that the wavefront is slowly varying over the aperture of a given lenslet. We can assume that the wavefront over a given lenslet is given by a tilted plane wave. The tilt is given by the local wavefront slope. A plane wave is focused to the rear focal plane of the lenslet. The focal spot, however, is displaced from the optical axis of the lenslet due to the tilt. By measuring

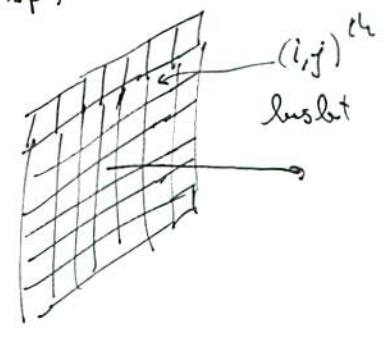
the spot displacements, the gradient (i.e. x and y derivatives) of the wavefront error can be obtained. In turn, the gradient can be numerically integrated to recover the wavefront error.



Consider the $(i, j)^{th}$ lenslet in the array. It is centered at a point (i_p, j_p)

$$\tan \Theta_y = \frac{\Delta y}{f}$$

but $\tan \Theta_y = -\frac{dw(i_p, j_p)}{dy}$



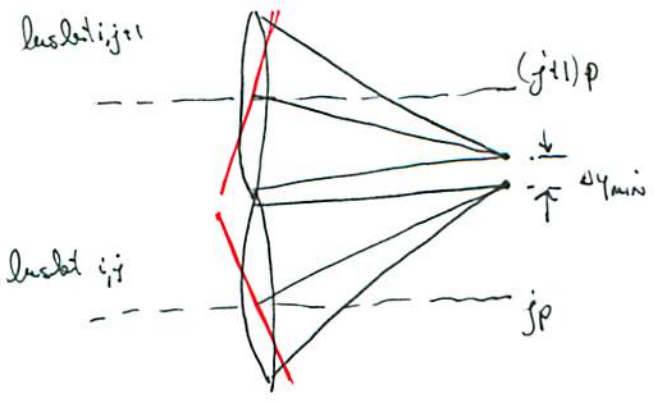
so $\Delta y_{ij} = -f \frac{dw}{dy}(i_p, j_p)$

Similarly $\Delta x_{ij} = -f \frac{dw}{dx}(i_p, j_p)$

Practically, we don't get perfect point but instead some finite size to the focal spot. In this case, the spot centroid is used. There is a maximum change in slope that can occur between two adjacent lenslets.

Dynamic Range
MAXIMUM SLOPE CHANGE BETWEEN LENSLETS

Spots can merge or cross over if slope change is too big between lenslets



Want $(j+1)p + \Delta y_{i,j+1} - [jp + \Delta y_{i,j}] \geq \Delta y_{min}$

where Δy_{min} is some minimum separation between adjacent spots

So,

$$p + \Delta y_{i,j+1} - \Delta y_{i,j} \geq \Delta y_{min}$$

$$p - f \frac{d\omega}{dy} (ip, (j+1)p) + f \frac{d\omega}{dy} (ip, jp) \geq \Delta y_{min}$$

$$\frac{d\omega}{dy_1} (ip, (j+1)p) - \frac{d\omega}{dy_1} (ip, jp) \leq \frac{p - \Delta y_{min}}{f}$$

MAXIMUM SLOPE CHANGE BETWEEN LENSLETS

$$\frac{\frac{d\omega}{dy_1} (ip, (j+1)p) - \frac{d\omega}{dy_1} (ip, jp)}{p} \approx \frac{d^2\omega}{dy_1^2} (ip, jp) \leq \frac{p - \Delta y_{min}}{pf}$$

MAXIMUM WAVEFRONT CURVATURE

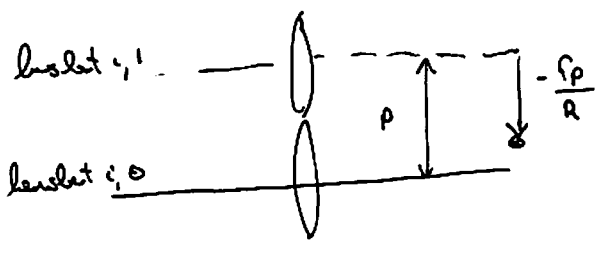
Simple Example w 1-D

$$w = \frac{y_1^2}{2R} \Rightarrow \frac{d\omega}{dy_1} = \frac{y_1}{R} \Rightarrow \frac{d^2\omega}{dy_1^2} = \frac{1}{R}$$

Suppose $\Delta y_{min} = 0$

$$\Delta y_{i,0} = 0$$

$$\Delta y_{i,1} = -\frac{fp}{R}$$



Maximum Slope Change Says

$$\frac{p}{R} - 0 \leq \frac{p}{f} \Rightarrow \frac{1}{R} \leq \frac{1}{f} \Rightarrow R \geq f$$

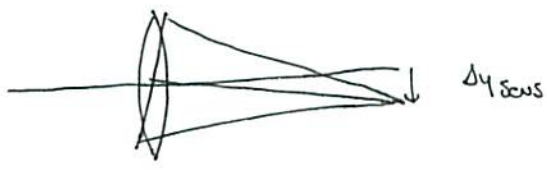
means $\Delta y_{i,1} \leq p$

Maximum Wavefront Curvature Says

$$\frac{1}{R} \leq \frac{1}{f} \quad \text{same thing}$$

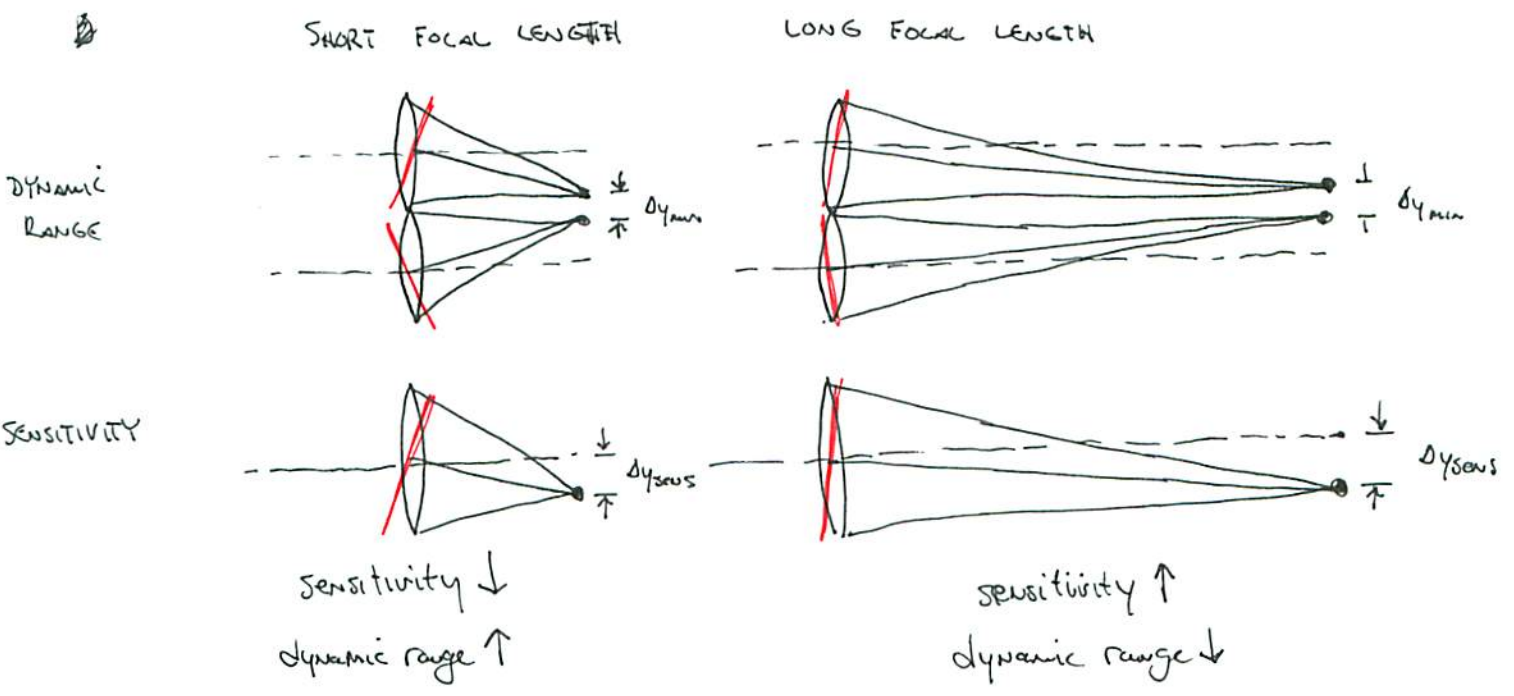
Practically, Δy_{min} will be limited by the pixel size on the detector or the size of the focal spot (like Rayleigh criterion how close can these spots get and still be resolved?)

Sensitivity - what's the smallest absolute slope that can be measured?



$$\Delta y_{sens} = -f \frac{dw_{min}}{dy_1}$$

$$\left| \frac{dw_{min}}{dy_1} \right| = \frac{\Delta y_{sens}}{f}$$



Effect of Relay System

$$w(x_0, y_0) = w(x_1, y_1) \quad \text{where} \quad x_1 = mx_0; \quad y_1 = my_0 \quad m = \text{relay magnification}$$

$$\frac{dx_1}{dx_0} = m; \quad \frac{dy_1}{dy_0} = m$$

chain rule

$$\frac{dw}{dx_0} = \frac{dw}{dx_1} \frac{dx_1}{dx_0} \Rightarrow$$

$$\frac{dw}{dx_1} = \frac{1}{m} \frac{dw}{dx_0}$$

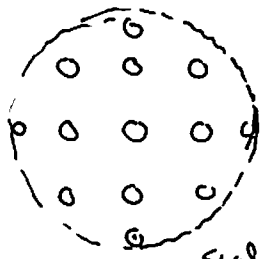
$$\frac{dw}{dy_1} = \frac{1}{m} \frac{dw}{dy_0}$$

Slope at lenslet

slope at E'

For $m < 1$, slope is steeper at lenslet away than at E'
Dynamic Range ↓

SHACK HARTMANN SPOT PATTERN



scaled exit pupil

$$\text{cyl} \left(\frac{r_1}{m \lambda f} \right) \sum_{i,j} \left[\delta \left(x_1 - ip + f \frac{d\omega}{dx_1}(ip, jp), y_1 - jp + f \frac{d\omega}{dy_1}(ip, jp) \right) * B(x_1, y_1) \right]$$

Spot shifts due to aberration

blur due to source size and diffraction

$$r_1^2 = x_1^2 + y_1^2$$

$$\text{cyl} \left(\frac{r_1}{m \lambda f} \right) \sum_{i,j} B \left(x_1 - ip + f \frac{d\omega}{dx_1}(ip, jp), y_1 - jp + f \frac{d\omega}{dy_1}(ip, jp) \right)$$

Centroid of $B(x_1, y_1)$ is a $(\frac{d_x}{dx_1}, \frac{d_y}{dy_1})$ where

$$d_x = \frac{\int x_1 B(x_1, y_1) dx_1 dy_1}{\int B(x_1, y_1) dx_1 dy_1}, \quad d_y = \frac{\int y_1 B(x_1, y_1) dx_1 dy_1}{\int B(x_1, y_1) dx_1 dy_1}$$

Centroids of the spots are given by

$$\text{cyl} \left(\frac{r_1}{m \lambda f} \right) \sum_{i,j} \delta \left(x_1 - ip + f \frac{d\omega}{dx_1}(ip, jp) - d_x, y_1 - jp + f \frac{d\omega}{dy_1}(ip, jp) - d_y \right)$$

Centroid of $B(x_1, y_1)$ shifts entire pattern by constant amount.

The preceding is only true for ~~interior~~ lenslets interior to the cyl() function. Spots that are clipped by the cyl() function will have displaced centroids and induce slight errors.

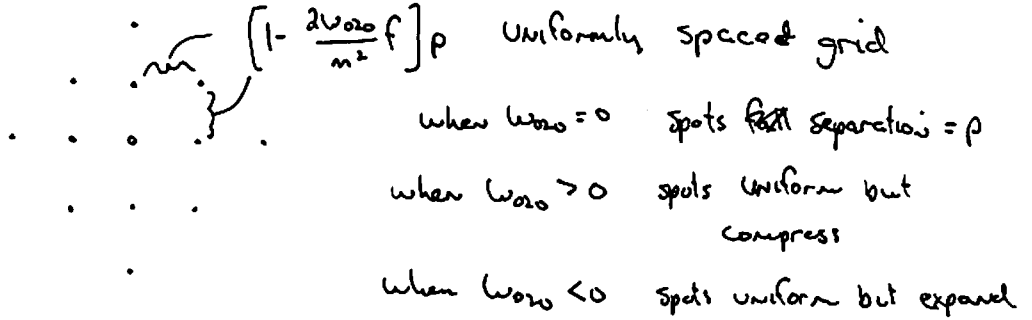
Example $\omega(x_0, y_0) = \omega_{00} (x_0^2 + y_0^2) \Rightarrow$ Defocus in Exit Pupil

$$\omega(x_1, y_1) = \frac{\omega_{00}}{m^2} (x_1^2 + y_1^2)$$

$$\frac{d\omega}{dx_1} = \frac{2\omega_{00}}{m^2} x_1, \quad \frac{d\omega}{dy_1} = \frac{2\omega_{00}}{m^2} y_1$$

SPAT PATTERN

$$\text{cyl} \left(\frac{r_i}{mD_{eff}} \right) \sum_{ij} \delta \left(x_i - \left(1 - \frac{d\omega_{020}}{m^2} f \right) i\rho - \bar{x}, y_i - \left(1 - \frac{d\omega_{020}}{m^2} f \right) j\rho - \bar{y} \right)$$



3.2.5.1 Fitting Shack-Hartmann Data to Zernike Polynomials

STEPS TO MEASURING A WAVEFRONT WITH A SHACK HARTMANN SYSTEM

- ① Calibrate system with a perfect plane wave

This gives a set of spots for the case where $w(x, y) = 0$
 Only need to do this once

- ② Measure the test system

This gives a set of spots that are displaced by the aberrations of the system

- ③ Measure the distance between the ideal and aberrated spots to get

$$\{ \Delta x_{ij}, \Delta y_{ij} \}_{ij}$$

- ④ Convert this set to $\left\{ \frac{dw}{dx} \Big|_{(i\rho, j\rho)}, \frac{dw}{dy} \Big|_{(i\rho, j\rho)} \right\}_{ij}$ with eqs. from pg. 129

Want to integrate these slopes to get $w(x, y)$ back

One way to do this is to fit to Zernike polynomials

Before we did $w(\bar{x}, \bar{y}) = \sum a_{nm} Z_n^m(\bar{x}, \bar{y})$

$$\frac{dw(\bar{x}, \bar{y})}{d\bar{x}} = \sum a_{nm} \frac{dZ_n^m(\bar{x}, \bar{y})}{d\bar{x}} \quad \frac{dw(\bar{x}, \bar{y})}{d\bar{y}} = \sum a_{nm} \frac{dZ_n^m(\bar{x}, \bar{y})}{d\bar{y}}$$

Set up a matrix system

$$\begin{pmatrix}
 \text{[Crossed out]} & \frac{dZ_1^i(\bar{x}_1, \bar{y}_1)}{d\bar{x}} & \dots & \frac{dZ_n^i(\bar{x}_1, \bar{y}_1)}{d\bar{x}} \\
 \vdots & \vdots & & \vdots \\
 \text{[Crossed out]} & \frac{dZ_1^i(\bar{x}_N, \bar{y}_N)}{d\bar{x}} & \dots & \frac{dZ_n^i(\bar{x}_N, \bar{y}_N)}{d\bar{x}} \\
 \text{[Crossed out]} & \frac{dZ_1^i(\bar{x}_1, \bar{y}_1)}{d\bar{y}} & \dots & \frac{dZ_n^i(\bar{x}_1, \bar{y}_1)}{d\bar{y}} \\
 \vdots & \vdots & & \vdots \\
 \text{[Crossed out]} & \frac{dZ_1^i(\bar{x}_N, \bar{y}_N)}{d\bar{y}} & \dots & \frac{dZ_n^i(\bar{x}_N, \bar{y}_N)}{d\bar{y}}
 \end{pmatrix}
 =
 \begin{pmatrix}
 a_{11} \\
 a_{12} \\
 \vdots \\
 a_{2n}
 \end{pmatrix}
 =
 \begin{pmatrix}
 \frac{dW(\bar{x}_1, \bar{y}_1)}{d\bar{x}} \\
 \frac{dW(\bar{x}_2, \bar{y}_2)}{d\bar{x}} \\
 \vdots \\
 \frac{dW(\bar{x}_N, \bar{y}_N)}{d\bar{x}} \\
 \frac{dW(\bar{x}_1, \bar{y}_1)}{d\bar{y}} \\
 \frac{dW(\bar{x}_2, \bar{y}_2)}{d\bar{y}} \\
 \vdots \\
 \frac{dW(\bar{x}_N, \bar{y}_N)}{d\bar{y}}
 \end{pmatrix}$$

Don't use $\frac{dZ_0^i}{dx}$ or $\frac{dZ_0^i}{dy}$ because it gives a column of zeros.

dN rows

$$D^T A = W$$

\bar{x}, \bar{y} Normalized coordinates

$$A = [D^T D]^{-1} D^T W$$

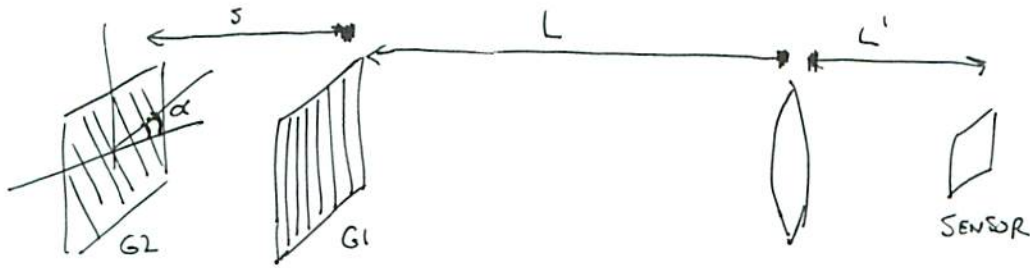
NOTE: The coordinates \bar{x}, \bar{y} are normalized coordinates

$$\bar{x} = \frac{x_i}{\left[\frac{mDe^i}{2} \right]} \quad \bar{y} = \frac{y_i}{\left[\frac{mDe^i}{2} \right]}$$

$$\frac{dZ_n^i}{d\bar{x}} = \frac{dZ_n^i}{dx} \frac{dx}{d\bar{x}} = \frac{mDe^i}{2} \frac{dZ_n^i}{dx}$$

Similar for $\frac{dZ_n^i}{d\bar{y}}$ and $\frac{dW}{d\bar{x}}, \frac{dW}{d\bar{y}}$

Show slides / demo



Moiré Deflectometry is a technique in which a Moiré pattern is created between two gratings G1 and G2 and imaged by a camera. A test lens can be placed in the path ~~to detect~~ and its power determined from the effect it has on the Moiré pattern.

$$\text{let } m_1 = \frac{L'}{L} \quad \text{magnification of } G1 \text{ on sensor}$$

$$m_2 = \frac{L'}{L+S} \quad \text{magnification of } G2 \text{ on sensor}$$

In sinusoidal gratings, the ^{patterns} transmissions of the gratings are given by

$$\begin{aligned} G1 &= \frac{1}{2} \left(1 + \cos \left(\frac{2\pi x}{m_1 g_1} \right) \right) \\ G2 &= \frac{1}{2} \left(1 + \cos \left(\frac{2\pi}{m_2 g_2} (x \cos \alpha + y \sin \alpha) \right) \right) \end{aligned} \quad \left. \vphantom{\begin{aligned} G1 \\ G2 \end{aligned}} \right\} \text{ on sensor plane.}$$

where α is rotation of G2 relative to G1

Perceptually, we see a "dark" fringe when both G1 and G2 are zero, and a "bright" fringe when $G1 = G2 = 1$. There is no interference here. To capture this effect, we multiply G1 and G2.

$$G1 \cdot G2 = \frac{1}{4} (1 + \cos \phi_1 + \cos \phi_2 + \cos \phi_1 \cos \phi_2)$$

$$\text{where } \phi_1 = \frac{2\pi x}{m_1 g_1} \quad \text{and} \quad \phi_2 = \frac{2\pi}{m_2 g_2} (x \cos \alpha + y \sin \alpha)$$

Let $\phi_1 = \frac{\alpha_0 + \beta_0}{2}$ and $\phi_2 = \frac{\alpha_0 - \beta_0}{2}$ where $\alpha_0 = \phi_1 + \phi_2$; $\beta_0 = \phi_1 - \phi_2$

and use $\frac{1}{2}(\cos \alpha_0 + \cos \beta_0) = \cos\left(\frac{\alpha_0 + \beta_0}{2}\right) \cos\left(\frac{\alpha_0 - \beta_0}{2}\right)$

$$G_1 \cdot G_2 = \frac{1}{4} \left[1 + \cos \phi_1 + \cos \phi_2 + \frac{1}{2} \cos(\phi_1 + \phi_2) + \frac{1}{2} \cos(\phi_1 - \phi_2) \right]$$

The last term represents the low frequency moiré fringes we perceive.

$$\phi_1 - \phi_2 = \cancel{2\pi} \left(\frac{1}{m_1 g_1} - \frac{\cos \alpha}{m_2 g_2} \right) x - \cancel{2\pi} \frac{\sin \alpha}{m_2 g_2} y = \cancel{2\pi} m \quad \begin{array}{l} m \text{ integer} \\ \text{for peaks} \end{array}$$

Equation of a line

$$y = \left[\frac{\frac{1}{m_1 g_1} - \frac{\cos \alpha}{m_2 g_2}}{\frac{\sin \alpha}{m_2 g_2}} \right] x - \frac{m m_2 g_2}{\sin \alpha}$$

The slope of this line is

$$\tan \Theta_0 = \frac{\frac{1}{m_1 g_1} - \frac{\cos \alpha}{m_2 g_2}}{\frac{\sin \alpha}{m_2 g_2}}$$

if $g_1 = g_2$

$$\tan \Theta_0 = \frac{\frac{1}{L+S} - \cos \alpha}{\sin \alpha}$$

What happens if we place a lens of power ϕ at G_1 ?

m_1 and g_1 are unchanged

m_2 and g_2 change.

The new G_2' is basically the image of G_2 through the test lens

Use image location for s or above and scale g_2 by the magnification

$$\frac{1}{s'} - \frac{1}{s} = \phi$$

$$\frac{1}{s'} = \frac{s\phi}{s} + \frac{1}{s}$$

$$s' = \frac{s}{1+s\phi} \quad m_s = \frac{s'}{s} = \frac{1}{1+s\phi}$$

So G2 appears to be at $s \rightarrow \frac{s}{1+s\phi}$ and $g_2 \rightarrow \frac{g_2}{1+s\phi}$

~~tan theta~~ $\frac{m_2 g_2}{m_1 g_1} \Rightarrow \frac{L}{L + \frac{s}{1+s\phi}} \cdot \frac{g_2}{1+s\phi} \cdot \frac{1}{g_1}$

$$\frac{m_2 g_2}{m_1 g_1} \Rightarrow \frac{L}{L(1+s\phi) + s} \frac{g_2}{g_1}$$

$$\tan \theta_0 = \frac{\frac{L}{L(1+s\phi) + s} \frac{g_2}{g_1} - \cos \alpha}{\sin \alpha}$$

SHOW MORE SLIDE WITH LENSES

FRINGES ROTATE AS FUNCTION OF ϕ

Can invert this function so that

$$\phi = \frac{1}{s(\sin \alpha \tan \theta_0 + \cos \alpha)} - \frac{1}{L} - \frac{1}{s}$$

POWER OF LENS GIVEN L, s, α and θ_0