4 Basic Inteferometry and Optical Testing

4.1 Review of Two Beam Interference

4.1.1 Plane Waves

$$E_1(\vec r,t) = A_1 \hat p_1 e^{i(\vec k_1 \cdot \vec r - \omega_1 t)}$$

$$E_2(\vec r,t) = A_2 \hat p_2 e^{i(\vec k_2 \cdot \vec r - \omega_2 t)}$$

$$A_1 = a_1 e^{i\phi_1} \begin{cases} a_1 : \text{amplitude} \\ \phi_1 : \text{phase} \end{cases}$$

$\vec k_1$ and $\vec k_2$ are propagation directions of the plane waves

$|\vec k_1| = \frac{2\pi N}{\lambda_1}$ where N is index of medium, λ_1 is wavelength

$\hat p_1 \cdot \vec k_1 = 0$ for transverse EM wave

similar for $\vec k_2$, $\hat p_2$

Combine these fields to find irradiance

$$I(\vec r,t) = \left|E_1(\vec r,t) + E_2(\vec r,t)\right|^2$$

$$I(\vec r,t) = |A_1|^2 + |A_2|^2 + A_1 A_2^* (\hat p_1 \cdot \hat p_2) e^{i[(\vec k_1 - \vec k_2)\cdot \vec r + (\omega_1 - \omega_2)t]} + A_1^* A_2 (\hat p_1 \cdot \hat p_2) e^{-i[(\vec k_1 - \vec k_2)\cdot \vec r + (\omega_1 - \omega_2)t]}$$

$$I(\vec r,t) = a_1^2 + a_2^2 + a_1 a_2 (\hat p_1 \cdot \hat p_2) e^{i[(\vec k_1 - \vec k_2)\cdot \vec r + (\omega_1 - \omega_2)t + \phi_1 - \phi_2]} + a_1 a_2 (\hat p_1 \cdot \hat p_2) e^{-i[(\vec k_1 - \vec k_2)\cdot \vec r + (\omega_1 - \omega_2)t + \phi_1 - \phi_2]}$$

define $I_1 = a_1^2$ $\quad I_2 = a_2^2$ $\quad \phi_1 - \phi_2 = \phi$

$$\boxed{I(\vec r,t) = I_1 + I_2 + 2\sqrt{I_1 I_2}\,(\hat p_1 \cdot \hat p_2) \cos\left((\vec k_1 - \vec k_2)\cdot \vec r + (\omega_1 - \omega_2)t + \phi\right)}$$

Usually we assume $\hat p_1 \cdot \hat p_2 = 1$ i.e. electric field is polarized in same direction

$\omega_1 - \omega_2 = 0$ i.e. same frequency

Under these assumptions

$$\boxed{I(\vec r) = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\left[(\vec k_1 - \vec k_2)\cdot \vec r + \phi\right]}$$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

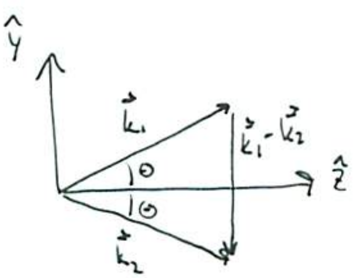
$$\text{Contrast}: \quad \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$$

What does this pattern look like?

Bright fringe when $(\vec{k}_1 - \vec{k}_2)\cdot\vec{r} + \phi = 2m\pi$ $\quad$ m integer

Dark fringe when $(\vec{k}_1 - \vec{k}_2)\cdot\vec{r} + \phi = (2m+1)\pi$

Let's align our coordinate system so that $\vec{k}_1$ and $\vec{k}_2$ are in the yz plane and orient the z axis so it bisects $\vec{k}_1$ and $\vec{k}_2$



$$\vec{k}_1 = \frac{2\pi n}{d}\left[\sin\theta\,\hat{y} + \cos\theta\,\hat{z}\right]$$

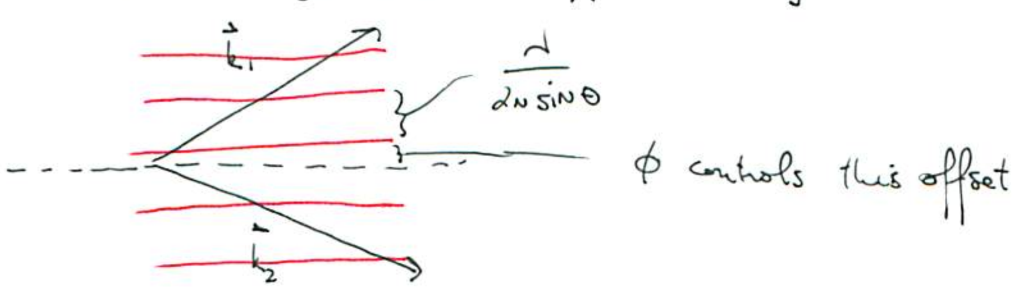
$$\vec{k}_2 = \frac{2\pi n}{d}\left[-\sin\theta\,\hat{y} + \cos\theta\,\hat{z}\right]$$

$$\vec{k}_1 - \vec{k}_2 = \frac{2\pi n}{d}\left[2\sin\theta\,\hat{y}\right]$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

So $(\vec{k}_1 - \vec{k}_2)\cdot\vec{r} = \frac{2\pi n}{d}\left[2y\sin\theta\right]$

Bright fringe when $\frac{2\pi n}{d}\left[2y\sin\theta\right] + \phi = 2m\pi$



4.1.3 General Wavefront Shapes

$$E_1(\vec{r},t) = A_1 e^{i\frac{2\pi}{\lambda}W_1(\vec{r})}$$

$$E_2(\vec{r},t) = A_2 e^{i\frac{2\pi}{\lambda}W_2(\vec{r})}$$

Again

$$I(\vec{r}) = \left|E_1(\vec{r},t) + E_2(\vec{r},t)\right|^2$$

$$\boxed{I(\vec{r}) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left[\frac{2\pi}{\lambda}(W_1 - W_2) + \phi\right]}$$

Bright fringes when $\frac{2\pi}{\lambda}(W_1 - W_2) + \phi = 2m\pi$

Interferometry basically takes an unknown wavefront W_1 (test wavefront) and a known wavefront W_2 (reference wavefront) and combines them as above. The fringe pattern created by the cosine term is analyzed to recover $W_1 - W_2$. Furthermore, we can control I_1, I_2 ~~and~~ to maximize fringe contrast. Finally, we can take multiple measurements with different values of ϕ to aid in recovering $W_1 - W_2$

Why is this useful?

Example 1

Suppose W_1 is the wavefront emerging from the exit pupil of an optical system and W_2 is a spherical wave corresponding to the reference sphere, the $W_1 - W_2$ equals the wavefront error as we have previously defined. Furthermore, various metrics such as wavefront variance and Strehl ratios are related to this value

Example 2

Suppose W_1 is the wavefront reflected from some spherical surface under test and W_2 is a spherical wavefront corresponding to the ideal shape of this surface. $W_1 - W_2$ represent the sag difference ~~errors~~ between the surfaces. This may tell you where to grind/polish more or if the part meets spec.

4.1.4 Visibility

Visibility is another name for contrast as we defined previously

$$\text{Visibility} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$$

Note that when $I_1 = I_2$

$$\text{Visibility} = \frac{\cancel{2}\sqrt{I_1^2}}{\cancel{2} I_1} = 1$$

High contrast when individual beams have equal irradiance.

4.1.5 COHERENCE AND POLARIZATION

we have ignored these effects by assuming $\omega_1 = \omega_2$ and ~~$\hat{p}_1 \parallel \hat{p}_2$~~ $\hat{p}_1 \cdot \hat{p}_2 = 1$. Let's look at the effects of these two. From page 138

$$I(\vec{r},t) = I_1 + I_2 + 2\sqrt{I_1 I_2}\,(\hat{p}_1 \cdot \hat{p}_2)\cos\left[(\vec{k}_1 - \vec{k}_2)\cdot\vec{r} + (\omega_1 - \omega_2)t + \phi\right]$$

The location of the fringes doesn't just depend on $\vec{r}$, but also on t. In other words, the fringes move with time. If $\omega_1 - \omega_2$ is too big, then we can't measure the fringe pattern since it is moving too fast.

The other issue is random fluctuations in ϕ with time. To get interference, we need the light to come from the same source. S~~that~~ The light is split into a test and reference portion and then recombined to interfere. By coming from the same source, ϕ_1 and ϕ_2 correlate making ϕ essentially constant. ~~But~~ We also need the times it takes to traverse the two paths to be similar otherwise the correlation between ϕ_1 and ϕ_2 breaks down.

~~Polarization affects the visibility~~

Coherence time $\Delta t \approx \frac{1}{\Delta\nu}$

where $\Delta\nu$ = ~~bandwidth~~ bandwidths of source

Coherence length $\Delta l = c\Delta t = \frac{c}{\Delta\nu}$

where c is speed of light

$$\Delta l \approx \frac{\bar{\lambda}^2}{\Delta\lambda} = \frac{(\text{mean } \lambda)^2}{(\text{wavelength band})}$$

4.1.2 Spherical Wave

$$E_1(\vec{r},t) = \frac{A_1}{|\vec{r}-\vec{r}_1|} e^{ik|\vec{r}-\vec{r}_1| \cancel{- i\omega t}}$$

$$E_2(\vec{r},t) = \frac{A_2}{|\vec{r}-\vec{r}_2|} e^{ik|\vec{r}-\vec{r}_2| \cancel{- i\omega t}}$$

again $A_1 = a_1 e^{i\phi_1}$

$A_2 = a_2 e^{i\phi_2}$

assume polarized in same direction and have same frequency

$$I(\vec{r},t) = \left| E_1(\vec{r},t) + E_2(\vec{r},t) \right|^2$$

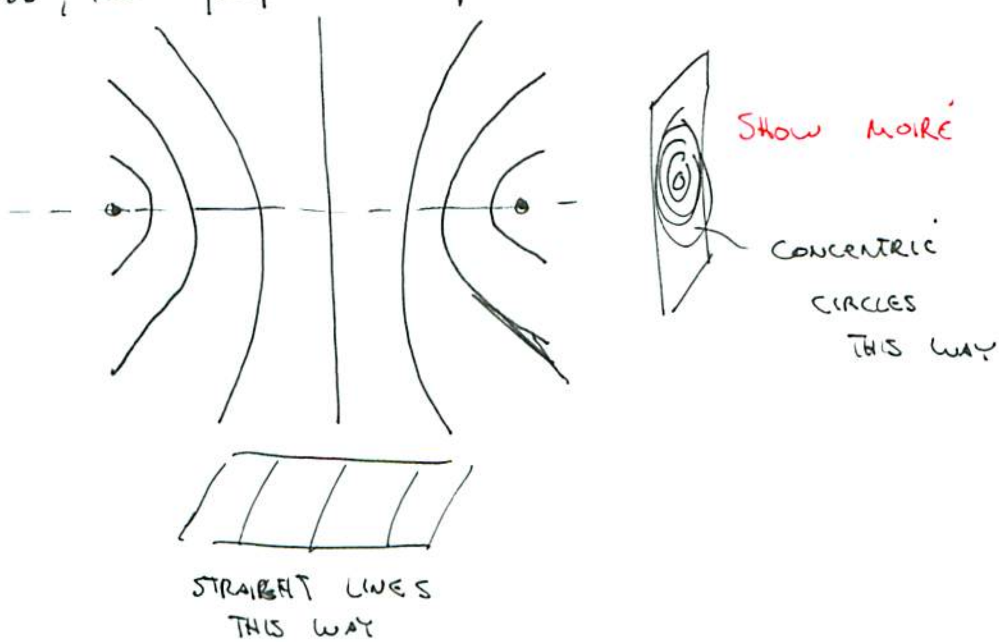
Following same procedure as before

$$I(\vec{r}) = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\left[\frac{2\pi n}{\lambda}\left(|\vec{r}-\vec{r}_1| - |\vec{r}-\vec{r}_2| \right) + \phi \right]$$

Bright fringes occur when

$$\frac{2\pi n}{\lambda}\left(|\vec{r}-\vec{r}_1| - |\vec{r}-\vec{r}_2| \right) + \phi \Big) = 2m\pi$$

In 3D, the equiphase surfaces are hyperboloids



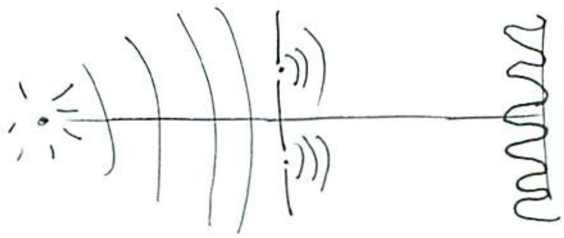
Polarization affects the visibility

$$\text{Visibility} = \frac{\alpha\sqrt{I_1 I_2}}{I_1 + I_2}\left(\hat{p}_1 \cdot \hat{p}_2\right)$$

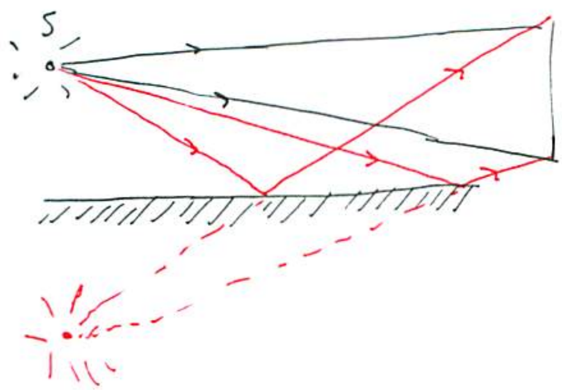
this is just cosine of $\measuredangle$ between the two vectors. When $\hat{p}_1 \perp \hat{p}_2$, the fringes have zero visibility

4.1.6 DIVISION OF WAVEFRONT

Spatially DIVIDE THE wavefront from a single source into two wavefronts and then recombine to get interference.

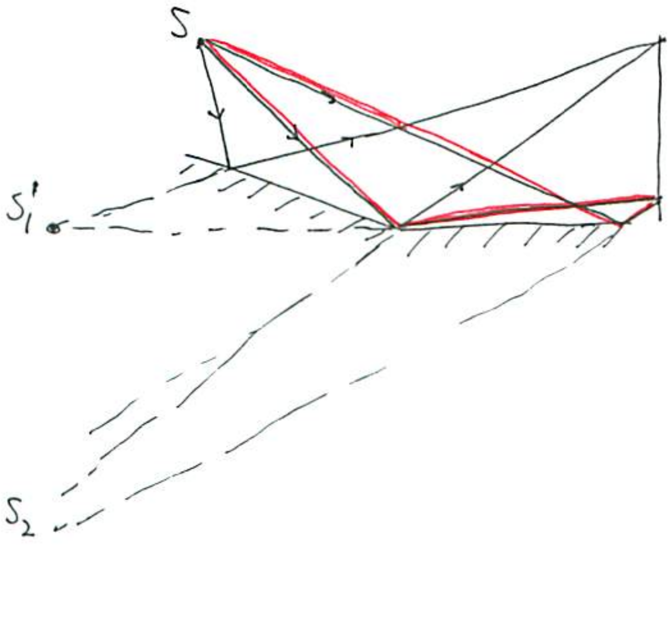


Young's Double Slit



Lloyd's Mirror

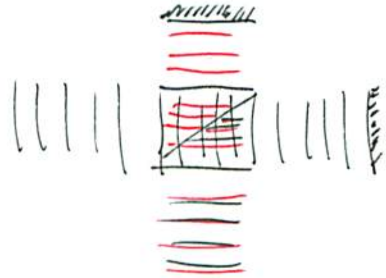
INTERFERENCE OF TWO SPHERICAL WAVES



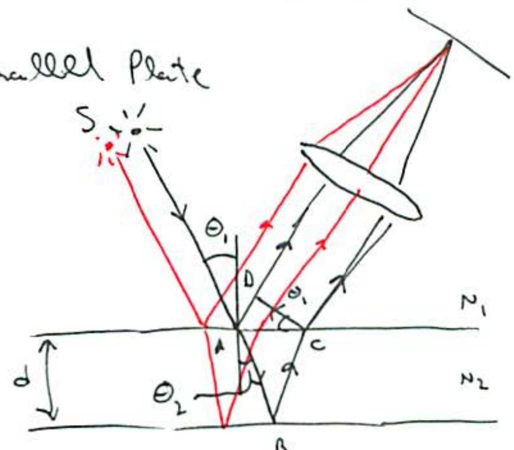
FRESNEL'S MIRRORS

4.1.7 DIVISION OF AMPLITUDE

USE A BEAMSPLITTER TO DIVIDE THE BEAM AND THE RECOMBINE



Plane Parallel Plate



From geometry

$$AB = BC = \frac{d}{\cos\theta_2}$$

$$AC = 2d\tan\theta_2$$

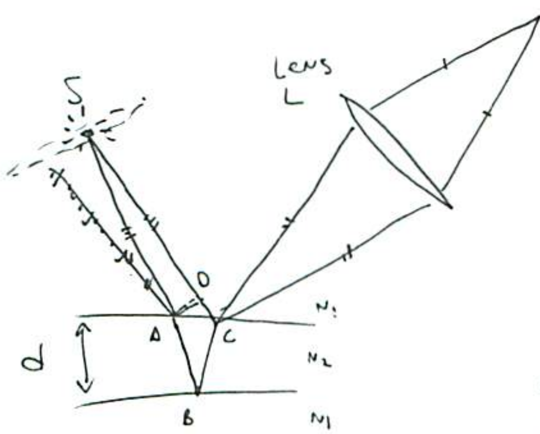
$$AD = AC\sin\theta_1 = 2d\tan\theta_2\sin\theta_1$$

$$OPD = n_2(AB+BC) - n_1 AD + \frac{\lambda}{2}$$

use $n_1\sin\theta_1 = n_2\sin\theta_2$

$$OPD = 2n_2 d\cos\theta_2 + \frac{\lambda}{2}$$

The $\frac{\lambda}{2}$ occurs because there is a π phase change upon reflection from only one of the interfaces (whenever we go from lower to higher index). These are known as fringes of equal inclination since extended source gives fringes at same location



Fringes of equal thickness

$$OPD \simeq 2n_2 d\cos\theta_2 + \frac{\lambda}{2}$$

Here the key is that we are imaging the surface boundary so the fringes are localized at the interface. In general, the fringe pattern changes with position on an extended source. For small d (i.e. a few waves) a large source can be used. However, as the thickness d increases the source must approach a point. The lens L can be the eye.

4.2 Newton's Rings

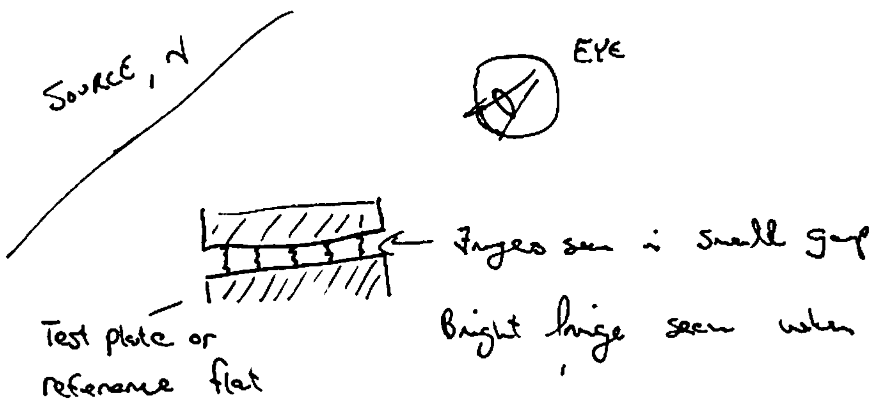
Newton's Rings are examples of fringes of equal thickness. One example is the color of fringe pattern seen from a thin oil film or bubble surface. Since the film is thin, a large source (sun, sky) can be used. Bright fringes occur when

$$OPD = 2n_2 d \cos\theta_2 + \frac{\lambda}{2} = m\lambda \qquad m \sim \text{integer}$$

$$2n_2 d \cos\theta_2 = (m - \tfrac{1}{2})\lambda$$

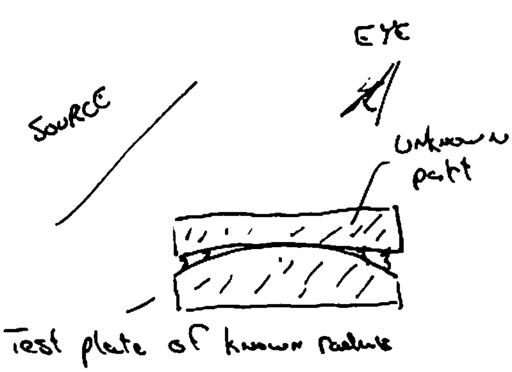
Different wavelengths correspond to different values of d, hence the color fringing effect.

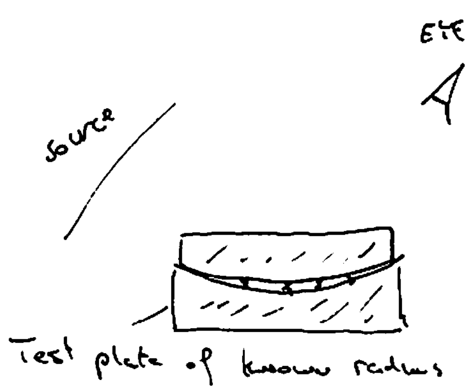
We can use this for optical testing as well. Consider ~~placing~~ testing how flat a surface is. If we place the test surface in contact with a KNOWN flat surface and the spacing is small, then fringes can be seen with an eye at the gap between the two surfaces. If we use a monochromatic source, the color fringe effect will be removed and the fringes will have higher contrast



Bright fringe seen when $2d\cos\theta_2 = (m - \tfrac{1}{2})\lambda$ since $n_2 = 1$ (i.e. air gap)

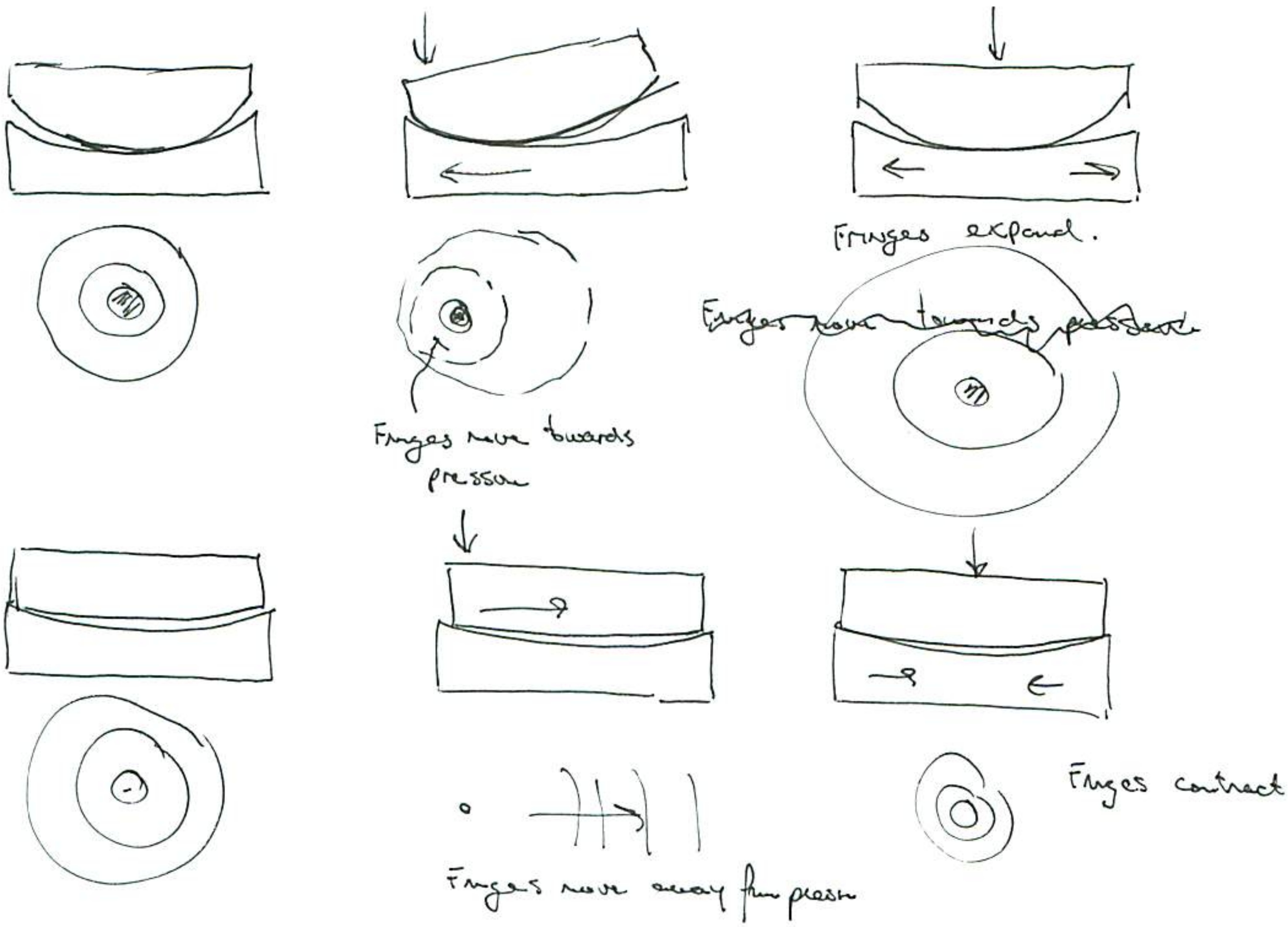
This technique can be extended to curved surfaces as well





In general, the test plate must be well characterized and at least as big as the unknown part. The parts must match pretty closely to see the fringes.

4.2.2 Determining if radius is shorter or longer than test plate



SHOW ~~[illegible]~~ PARK'S VIDEO

SHOW DEMO

4.2.3 TEST PLATES ~~[illegible]~~

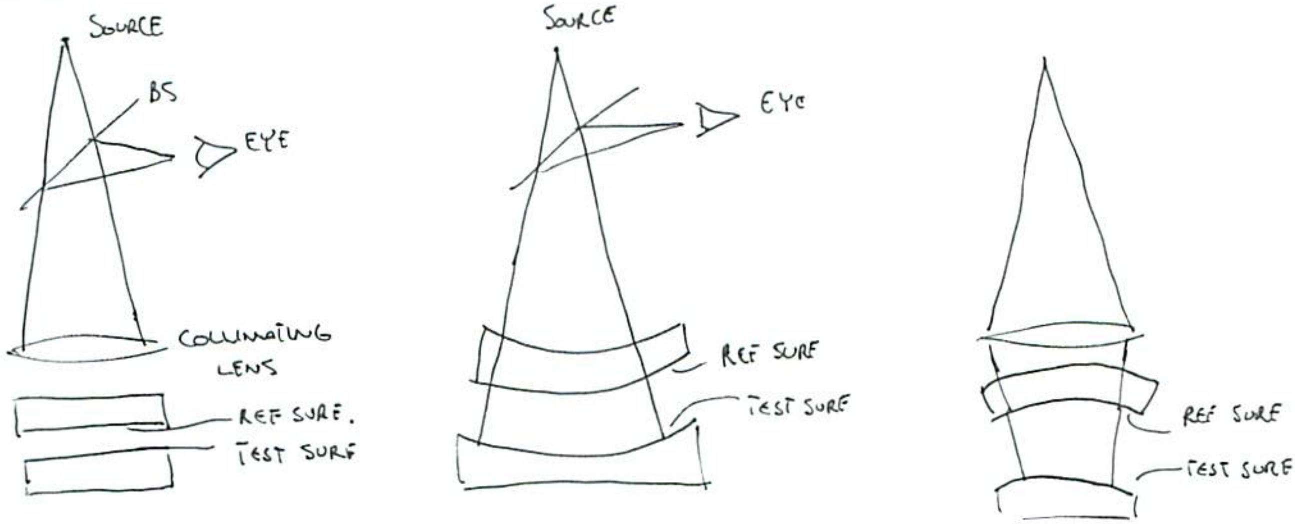
SHOW opcolab.com/page200.html

Most manufacturers have sets of test plates with well characterized radii. When ordering lenses from such a company you may specify test plate fits, but your design must be close to available test plates. Alternatively, test plates can be made for your part, but this is expensive and time consuming. Going this route is dictated by budget, timeframe and # of parts to be made/tested. It may be worthwhile to modify your design to better match available test plates.

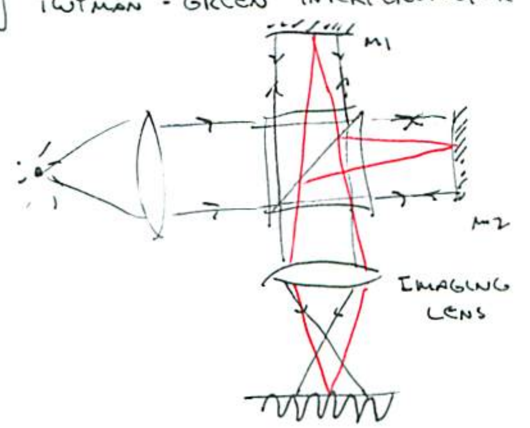
4.3 FIZEAU INTERFEROMETER

If gap between the reference and test surfaces becomes too large, or concerns regarding putting surfaces in contact then a ~~figeau~~ Fizeau interferometer can be used. A small monochromatic source is needed in this case.

4.3.1 COMMON CONFIGURATIONS

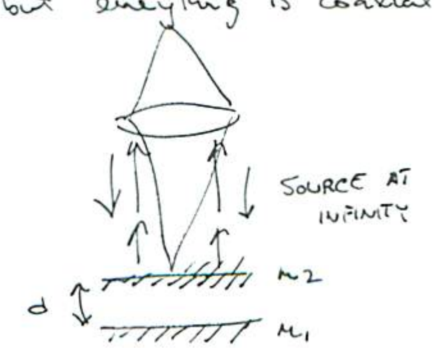


4.4 TWYMAN - GREEN INTERFEROMETER



The Twyman - GREEN uses a collimated monochromatic source. The imaging lens relays mirrors M1 and M2 to a common plane where the interference pattern is observed.

If we unfold the optical paths, we essentially have our plane parallel plate system but everything is coaxial

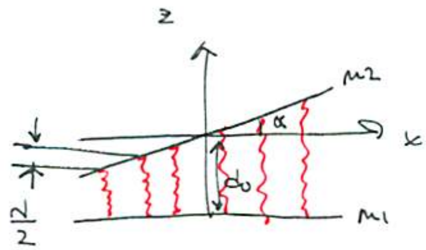


Fringes of equal thickness located in "gap" between M1 and M2. Main difference is that there isn't a π ~~ph~~ phase shift at one of the surfaces.

$$OPD = 2d \Rightarrow \text{bright fringe when } 2d = m\lambda$$
$$d = m\frac{\lambda}{2}$$

If we tip M1 with respect to M2, we get straight line fringes

$d(x) = d_0 +$

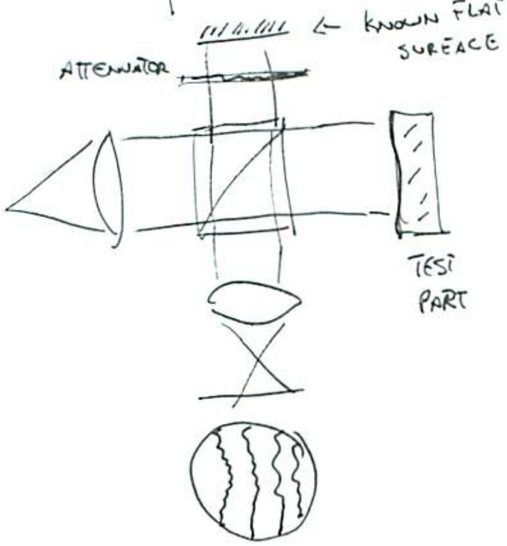


$d(x) : d_0 + x \tan\alpha$

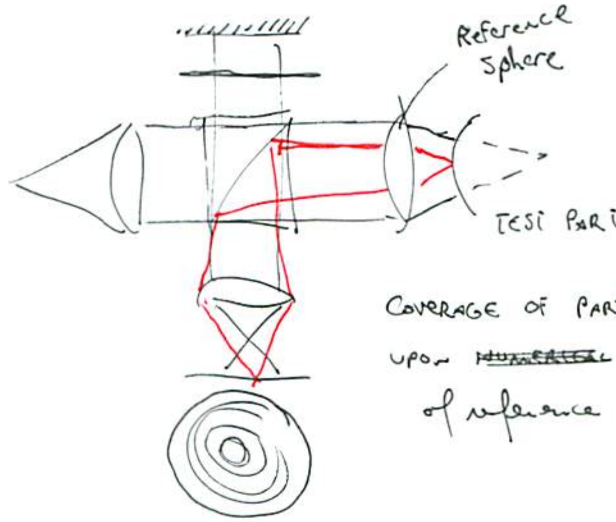
So fringes when $d_0 + x\tan\alpha = \frac{m\lambda}{2}$

If the fringes are not straight lines then one (or both) mirrors are not flat

Testing flat surfaces

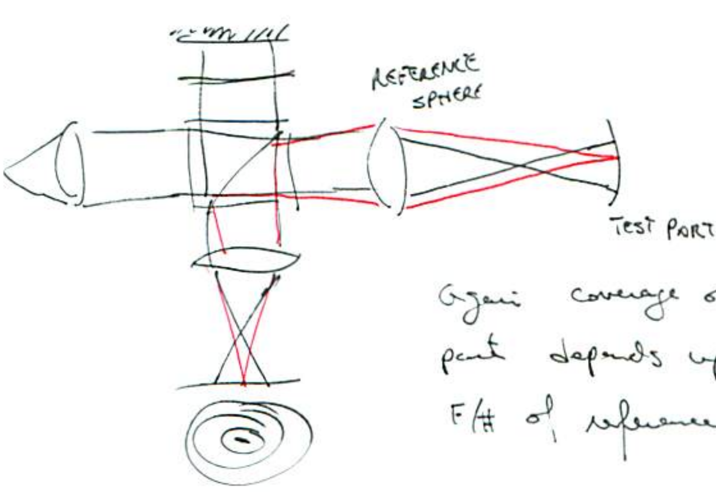


Testing Convex Part



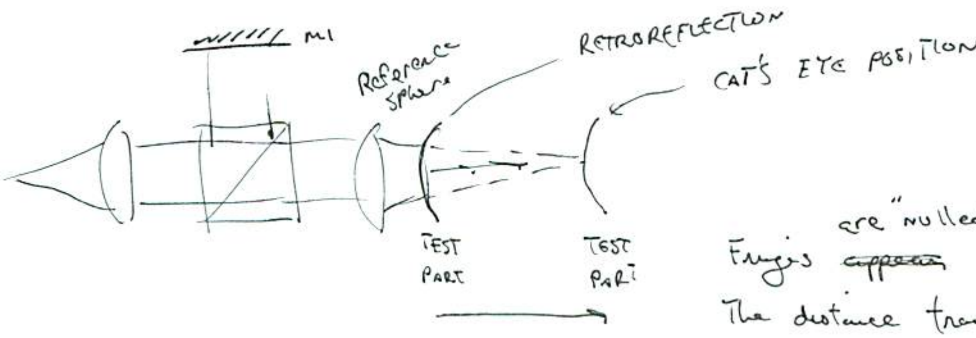
COVERAGE OF PART DEPENDS UPON ~~NUMERICAL~~ F/# of reference sphere.

Testing Concave Part



Again coverage of part depends upon F/# of reference sphere

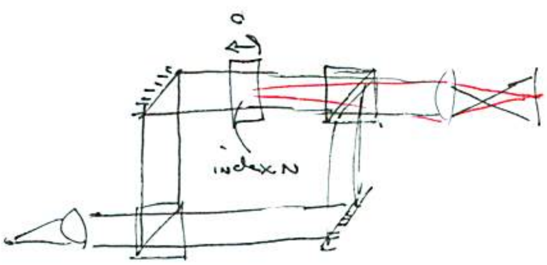
Measuring Radius of curvature



Fringes ~~appear~~ are "nulled" at the two positions. The distance traveled is the radius of the test part.

Fringes are "nulled" when plane wave from M1 and recollimated reflection from test part are parallel.

4.5 MACH-ZEHNDER INTERFEROMETER



Measures parts in transmission

OPD = Nd

BRIGHT FRINGE WHEN $Nd = m\lambda$

EXAMPLE OF USE: SUPPOSE WANT TO MEASURE UNIFORMITY OF REFRACTIVE

CUT CONSTANT THICKNESS SLAB OF GLASS, THEN FRINGE PATTERN DEPENDS ON INDEX vs. position

EXAMPLE: MEASURING A WINDOW

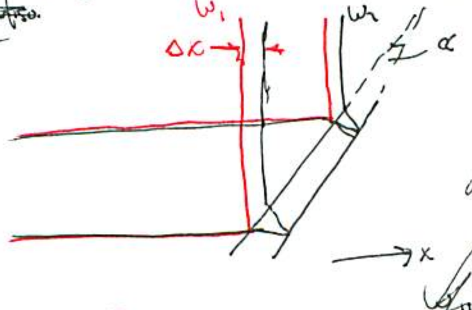
4.6 LATERAL SHEARING INTERFEROMETERS

Split wavefront $W(x,y)$ and displace it by Δx. Combine displaced and centered wavefront to get interference pattern

4.6.1 Common ~~Compo~~ Config

Shear plate

$$OPD = W_1(x,y) - W_2(x+\Delta x, y) \text{ proportional to } \frac{dW}{dx}$$



~~Defocus~~

~~$W = W_{020}(x^2+y^2) + W_{111}y$~~

~~$OPD = 2W_{020}\,x\,\Delta x + W_{111}y$~~

~~eq. of line~~

~~W_{111} introduced by wedge α~~

SHOW VIDEO

In the case of a shear plate

$$W_1(x,y) = W_{020}(x^2+y^2)$$

$$W_2(x+\Delta x, y) = W_{020}\left((x+\Delta x)^2 + y^2\right) + W_{111}y$$

So

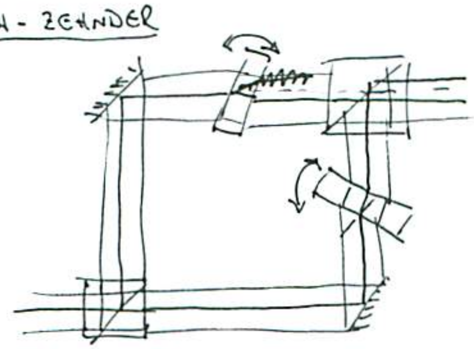
$$OPD = W_1(x,y) - W_2(x+\Delta x, y) = W_{020}(\cancel{x^2}+\cancel{y^2}) - W_{020}(\cancel{x^2} + 2\Delta x \cdot x + \Delta x^2 + \cancel{y^2}) - W_{111}y$$

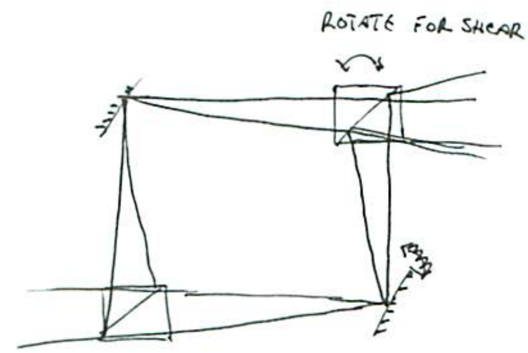
$$OPD = -2W_{020}\Delta x \cdot x - W_{020}\Delta x^2 - W_{111}y = m\lambda$$ for bright fringe, m = integer

$$y = \frac{-2W_{020}\Delta x}{W_{111}}x \quad \frac{-W_{020}\Delta x^2 - m\lambda}{W_{111}}$$ Eq. of a line

$$\text{Slope} = \frac{-2W_{020}\Delta x}{W_{111}}$$

MACH - ZEHNDER





Rotate plane parallel plates to give shear

4.7 Interferograms

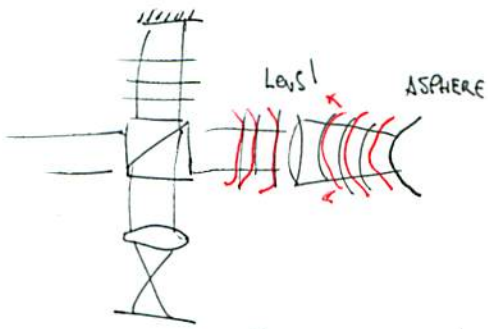
SHOW SLIDES

Visibility decrease with differences in beam intensities, but not that fast

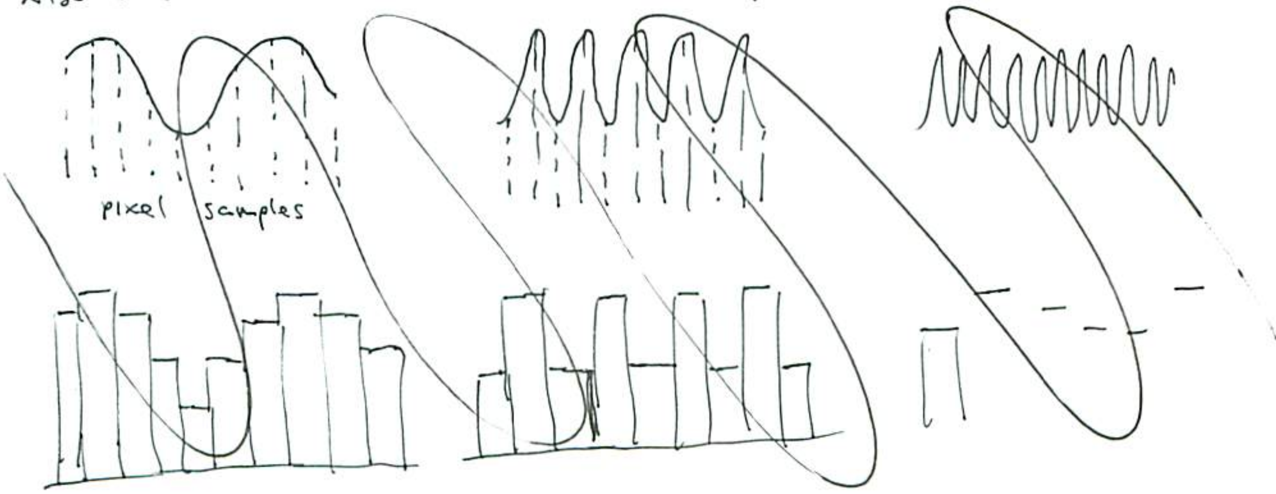
suppose $I_2 = 0.04\, I_1$ roughly glass/mirror comparison

$$\text{Visibility} = \frac{2\sqrt{0.04}\,\cancel{I_1}}{\cancel{I_1}(1.04)} = 0.385$$

Aliasing occurs when fringes get too close together



Testing aspheres are especially prone to aliasing due to the inability to easily match the aspheric wavefront on the reference wavefront. Also Lens 1 introduces aberrations because of asymmetric pass.



4.5 PHASE SHIFTING INTEROMETRY

Let's examine our basic interferometry expression

$$I(x,y) = I_1(x,y) + I_2(x,y) + 2\sqrt{I_1(x,y)\, I_2(x,y)}\, \cos\left[\frac{2\pi}{\lambda}\left(\omega_1(x,y) - \omega_2(x,y)\right) + \phi \right]$$

Let's rewrite this as

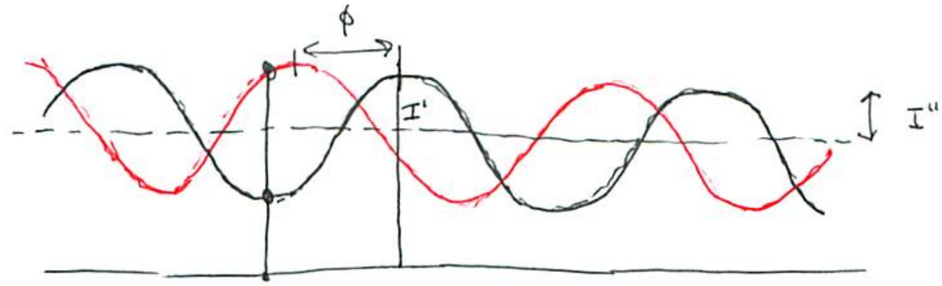
$$I(x,y) = I'(x,y) + I''(x,y) \cos\left[\Psi(x,y) + \phi \right]$$

where $I'(x,y) = I_1(x,y) + I_2(x,y)$

$$I''(x,y) = 2\sqrt{I_1(x,y)\, I_2(x,y)}$$

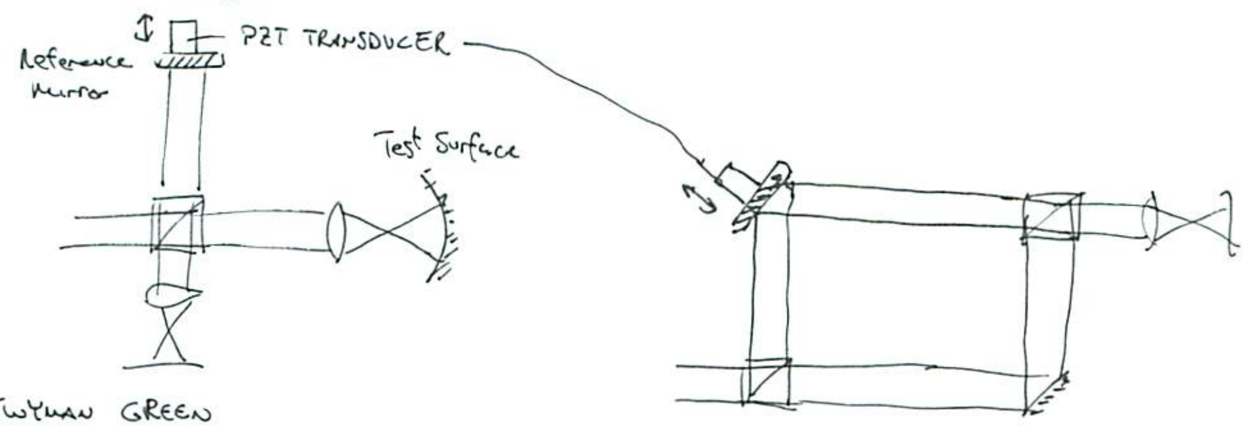
$$\Psi(x,y) = \frac{2\pi}{\lambda}\left(\omega_1(x,y) - \omega_2(x,y)\right)$$

For any given point (x_0, y_0), there are 3 unknowns $I'(x_0,y_0)$, $I''(x_0,y_0)$ and $\Psi(x_0,y_0)$. If we make at least 3 measurements with different values of ϕ, then we can solve exactly for I', I'' and Ψ. Furthermore, this holds for every pixel in the interferogram



ϕ shifts the cosine pattern. A given pixel sees different intensity values as a function of ϕ. The pattern repeats for ϕ equal to multiples of 2π.

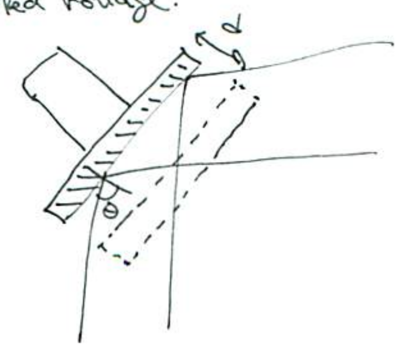
How to change ϕ between measurements



TWYMAN GREEN

MACH ZEHNDER

PZT = Piezoelectric Transducer - ceramic that expands and contracts with applied voltage.



wavefront displacement = $2d\cos\theta$

phase shift = $\frac{4\pi}{\lambda} d\cos\theta$

For simplicity let's start with four measurements with

$$\phi = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

$$I_1(x,y) = I'(x,y) + I''(x,y) \cos[\psi(x,y)]$$

$$I_2(x,y) = I'(x,y) + I''(x,y) \cos\left[\psi(x,y) + \frac{\pi}{2}\right]$$

$$I_3(x,y) = I'(x,y) + I''(x,y) \cos[\psi(x,y) + \pi]$$

$$I_4(x,y) = I'(x,y) + I''(x,y) \cos\left[\psi(x,y) + \frac{3\pi}{2}\right]$$

Using our trig identities, this reduces to

$$I_1 = I' + I'' \cos[\psi]$$ drop (x,y) for now

$$I_2 = I' - I'' \sin[\psi]$$

$$I_3 = I' - I'' \cos[\psi]$$

$$I_4 = I' + I'' \sin[\psi]$$

Note

$$I_4 - I_2 = 2 I'' \sin[\psi]$$

$$I_1 - I_3 = 2 I'' \cos[\psi]$$

Taking the ratio

$$\frac{I_4 - I_2}{I_1 - I_3} = \tan\psi \Rightarrow \boxed{\psi(x,y) = \tan^{-1}\left[\frac{I_4 - I_2}{I_1 - I_3}\right]}$$

This can be related back to OPD as well

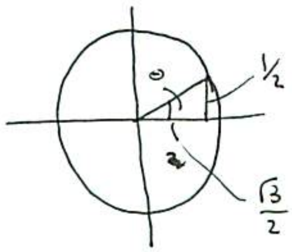
$$OPD(x,y) = \frac{\lambda}{2\pi} \psi(x,y)$$

Recall that Visibility $= \dfrac{2\sqrt{I_1 I_2}}{I_1 + I_2} = \dfrac{I''(x,y)}{I'(x,y)}$

$$\boxed{\text{Visibility} = \frac{2\sqrt{(I_4 - I_2)^2 + (I_1 - I_3)^2}}{I_1 + I_2 + I_3 + I_4}}$$

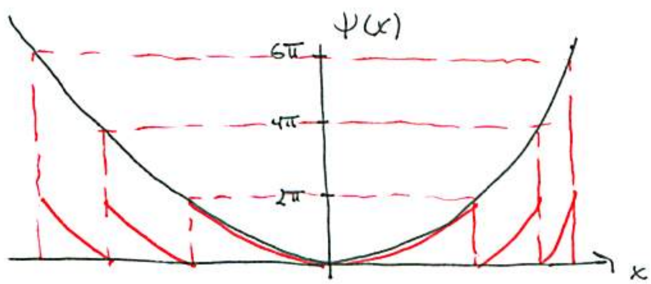
Holds for all pixels (x,y)

WRAPPED PHASE



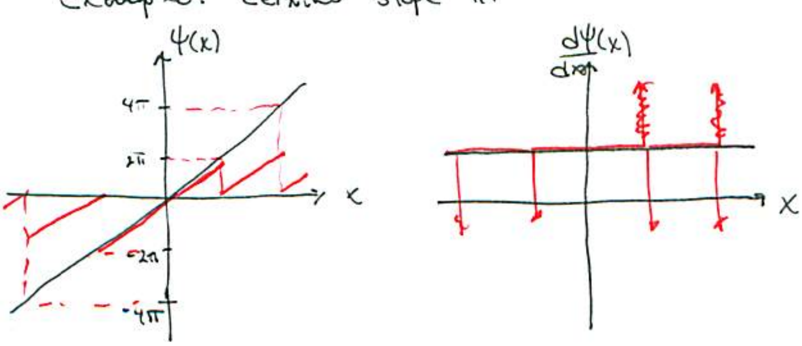
What's $\tan^{-1}\left[\dfrac{1/2}{\sqrt{3}/2}\right] = \ldots -330°,\ \underline{\underline{30°}},\ 390° \ldots$

The problem with $\psi(x,y) = \tan^{-1}\left[\dfrac{I_4 - I_2}{I_1 - I_3}\right]$ is that we don't know how many 360°s (2πs) to add or subtract.



Phase Unwrapping Algorithms - These are a general class of algorithms which take the wrapped phase and tries to unwrap it to recover the true, smooth $\psi(x,y)$. Conceptually, find every place with a discontinuity and add 2πs until the surface is smooth. In practice, for discrete data with noise, this problem gets more complex. Many algorithms exist.

Example: Zernike slope fit



① Approximate slope as $\dfrac{\psi(x) - \psi(x+\Delta x)}{\Delta x}$

② Fit slope to Zernike derivatives

$$\frac{d\psi(x)}{dx} = \sum a_{nm} \frac{dZ_n^m}{dx}$$

③ $\psi^{(1)}(x) = \sum a_{nm} Z_n^m$

④ $\varepsilon(x) = \psi(x) - \psi^{(1)}(x)$

⑤ Repeat ②-④ with $\varepsilon(x)$

$$\psi(x) \approx \psi^{(1)}(x) + \psi^{(2)}(x) + \ldots \psi^{(n)}(x)$$

Three Step Algorithm

$$\phi : -\alpha, 0, \alpha$$

$$\psi(x,y) : \tan^{-1}\left[\left(\frac{1-\cos\alpha}{\sin\alpha}\right)\frac{I_1 - I_3}{2I_2 - I_1 - I_3}\right]$$