OPTI 435/535 Homework #3 Due: October 3, 2011 Before Class

Undergrads do problems 1 through 3
Grads do all four problems

1. The mean wavefront error over a normalized pupil is given by

$$\overline{W} = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 W(\rho,\theta)\rho d\rho d\theta$$

and the wavefront variance is given by

$$\sigma_W^2 = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 \left(W(\rho,\theta)-\overline{W}\right)^2 \rho d\rho d\theta$$

where W(ρ,θ) is the wavefront error. For the following wavefront, show that the wavefront variance is just the sum of the squares of the Zernike expansion coefficients a_{nm}.

$$W(\rho,\theta) = a_{2,2}Z_2^2(\rho,\theta) + a_{3,1}Z_3^1(\rho,\theta)$$

Problem 1

Write out the Zernike functions explicitly

```
W[ρ_, θ_] = a22 * Sqrt[6] * ρ^2 * Cos[2 * θ] + a31 * Sqrt[8] * (3 * ρ^3 - 2 * ρ) * Cos[θ]
```

$2\sqrt{2}\ a31\ (-2\rho + 3\rho^3)\ Cos[\theta] + \sqrt{6}\ a22\ \rho^2\ Cos[2\theta]$

Next calculate the mean wavefront error by evaluating the integral

```
Wmean = (1 / π) * ∫_0^(2*π) (∫_0^1 W[ρ, θ] * ρ dρ) dθ
```

0

In general, the mean of all of the Zernike polynomials, except Z (0, 0) is zero. Next, calculate the wavefront variance by evaluating

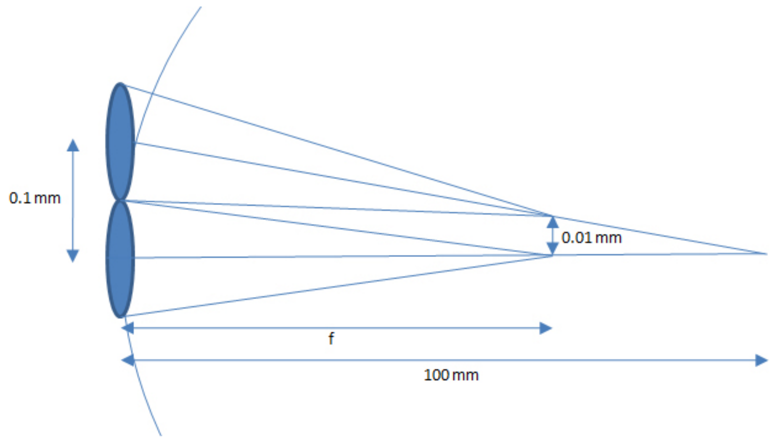
```
Wvariance = (1 / π) * ∫_0^(2*π) (∫_0^1 (W[ρ, θ] - Wmean) ^2 * ρ dρ) dθ
```

$a22^2 + a31^2$

which is the answer we were looking for.

2. The spot pattern from a Shack Hartmann wavefront sensor contracts with myopia. Suppose we have a lenslet array with lenslet spacing of 100 microns and our sensor can accurately separate spots that are spaced by 10 microns. What is the required focal length of the lenslets, if we wish to be able to measure up to 10 diopters of myopia?

Problem 2



A wavefront with 10 D of myopia will converge 1/10 D = 100 mm from the lenslet array. Each lenslet is spaced 0.1 mm and the minimum resolution of the sensor is 0.01 mm. The required focal length can be obtained from using similar triangles and the figure above.

```
Solve[0.1 / 100 == 0.01 / (100 - f), f]
{{f → 90.}}
```

3. A wavefront of the form $W = -0.002x^2$ is measured with a Shack Hartmann sensor for a 4 mm diameter pupil. Suppose the lenslets of the array have a focal length of 24 mm and a spacing of 1 mm.

(a) What does the *unaberrated* Shack Hartmann pattern look like?
(b) What are the focal spot shifts Δx and Δy for each spot?
(c) What does the Shack Hartmann pattern look like for the wavefront W?

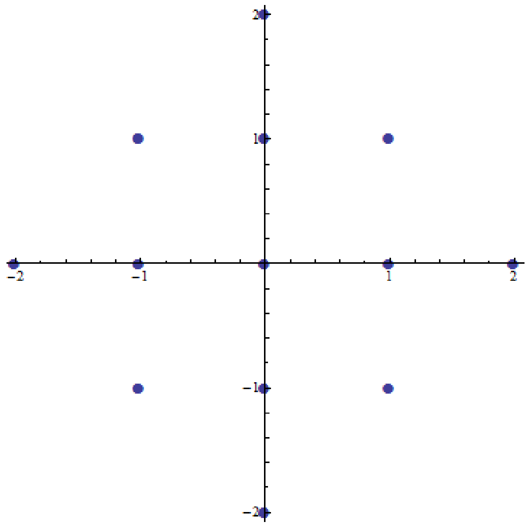
Problem 3

For a 1 mm spacing between the lenslets only the following coordinates pass through the pupil

```
points = {{-2, 0}, {-1, 0}, {0, 0}, {1, 0}, {2, 0}, {-1, 1}, {0, 1}, {1, 1}, {0, 2}, {-1, -1}, {0, -1}, {1, -1}, {0, -2}}
{{-2, 0}, {-1, 0}, {0, 0}, {1, 0}, {2, 0}, {-1, 1}, {0, 1}, {1, 1}, {0, 2}, {-1, -1}, {0, -1}, {1, -1}, {0, -2}}
```

The unaberrated spot pattern looks like

```
ListPlot[points, AspectRatio → 1, PlotMarkers → {Automatic, Medium}]
```



The spot shifts are given by $\Delta x = -f \frac{dW}{dx}$ and $\Delta y = -f \frac{dW}{dy}$. For $f = 24$ mm and $W = -0.002\, x^2$

```
Δx = -f * D[-0.002 * x^2, x]
Δy = -f * D[-0.002 * x^2, y]
0.004 f x
0
```

So Δy always equals zero and Δx is only dependent on the x position. Possible values for Δx are therefore

```
Δx /. {f → 24, x → -2}
Δx /. {f → 24, x → -1}
Δx /. {f → 24, x → 0}
Δx /. {f → 24, x → 1}
Δx /. {f → 24, x → 2}
```

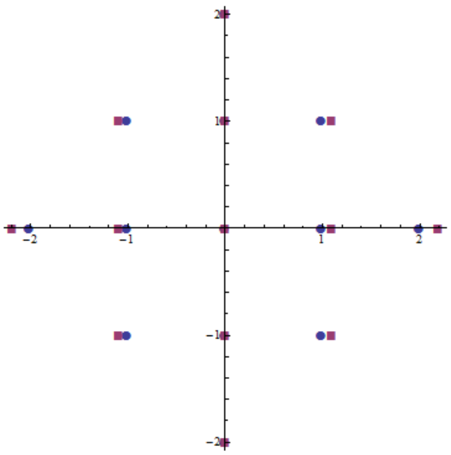
```
-0.192

-0.096

0

0.096

0.192
```

The aberrated spots are then located at

```
shiftedpoints = {{-2 - 0.192, 0}, {-1 - 0.096, 0}, {0, 0}, {1 + 0.096, 0}, {2 + 0.192, 0}, {-1 - 0.096, 1}, {0, 1}, {1 + 0.096, 1},
  {0, 2}, {-1 - 0.096, -1}, {0, -1}, {1 + 0.096, -1}, {0, -2}}
```

```
{{-2.192, 0}, {-1.096, 0}, {0, 0}, {1.096, 0}, {2.192, 0}, {-1.096, 1}, {0, 1}, {1.096, 1}, {0, 2}, {-1.096, -1}, {0, -1}, {1.096, -1}, {0, -2}}
```

```
ListPlot[{points, shiftedpoints}, AspectRatio → 1, PlotMarkers → {Automatic, Medium}]
```



*********************************Grads Only**********************************

4. Write the wavefront from Problem 3 in terms of Zernike polynomials.

First, the normalization radius r_{max} = 2mm (i.e. half the pupil diameter). Next, let's just consider the x^2 term in polar coordinates. This is given by

$$x^2 = r^2 \cos^2\theta = 4\rho^2 \cos^2\theta \text{ since } \rho = r / r_{max}.$$

Using the trig relation $\cos^2\theta = \frac{1}{2}[\cos 2\theta + 1]$ gives

$$x^2 = 2\rho^2 \cos 2\theta + 2\rho^2 = \frac{2}{\sqrt{6}} Z_2^2(\rho,\theta) + \frac{1}{\sqrt{3}} Z_2^0(\rho,\theta) + Z_0^0(\rho,\theta).$$

So,

$$W = -0.002\left[\frac{2}{\sqrt{6}} Z_2^2(\rho,\theta) + \frac{1}{\sqrt{3}} Z_2^0(\rho,\theta) + Z_0^0(\rho,\theta)\right].$$