Homework 4 Solutions

Questions 1

Suppose an eye has a wavefront error

$W(\rho, \theta) = 0.001\ Z_4^0\ (\rho, \theta)$

where $\rho = r / r_{max}$ and r_{max} = 3 mm. If the pupil shrinks to r_{max} = 1.5 mm, what is the new wavefront error in terms of Zernike polynomials?

In[1]:= `W[ρ_, θ_] = (1 / 1000) * Sqrt[5] * (6 ρ^4 - 6 ρ^2 + 1)`

Out[1]= $$\frac{1 - 6\rho^2 + 6\rho^4}{200\sqrt{5}}$$

Convert to unnormalized coordinates

In[3]:= `W[r / 3, θ]`

Out[3]= $$\frac{1 - \frac{2 r^2}{3} + \frac{2 r^4}{27}}{200\sqrt{5}}$$

For the new pupil size $\rho = r / 1.5, \rightarrow\ r = 1.5\rho$

In[4]:= `W[r / 3, θ] /. r → 3 ρ / 2`

Out[4]= $$\frac{1 - \frac{3\rho^2}{2} + \frac{3\rho^4}{8}}{200\sqrt{5}}$$

Now project this wavefront onto the various Zernike terms to get the coefficients. The coefficient for $Z_4^0$is

In[11]:= $$2\int_0^1 \left(\frac{1 - \frac{3\rho^2}{2} + \frac{3\rho^4}{8}}{200\sqrt{5}}\right) * \text{Sqrt}[5] * (6\rho\text{^}4 - 6\rho\text{^}2 + 1)\ \rho\, d\rho$$

Out[11]= $$\frac{1}{16\,000}$$

The coefficient for $Z_2^0$is

In[12]:= $$2\int_0^1 \left(\frac{1 - \frac{3\rho^2}{2} + \frac{3\rho^4}{8}}{200\sqrt{5}}\right) * \text{Sqrt}[3] * (2\rho\text{^}2 - 1)\ \rho\, d\rho$$

Out[12]= $$-\frac{3\sqrt{\frac{3}{5}}}{3200}$$

The coefficient for Z_0^0 is

In[13]:= $$2\int_0^1 \left(\frac{1-\frac{3\rho^2}{2}+\frac{3\rho^4}{8}}{200\sqrt{5}}\right)\rho\, d\rho$$

Out[13]= $$\frac{3}{1600\sqrt{5}}$$

Question 2

The power of the cornea is given by

```
ϕ1 = 1000 * (1.336 - 1) / 7.8
43.0769
```

For an object at infinity, the cornea (by itself) forms an image

```
Solve[1.336 / L1p == ϕ1, L1p]
{{L1p → 0.0310143}}
L1p = 0.031014285714285706`
0.0310143
```

We can then use the imaging equation do determine where the IOL sits

```
Solve[1.336 / (0.024 - d) - 1.336 / (L1p - d) == 20.0, d]
{{d → 0.00557875}, {d → 0.0494355}}
```

Choose the first solution since it is in the eye. So the IOL sits 5.58 mm behind the cornea.

For the object at 33cm, the cornea (by itself) forms an image at

```
Clear[L1p]
Solve[1.336 / L1p + 1 / .33 == ϕ1, L1p]
{{L1p → 0.0333611}}
L1p = 0.033361117578579735`
0.0333611
```

Calculate the IOL position d for the new object distance

```
Solve[1.336 / (0.024 - d) - 1.336 / (L1p - d) == 20.0, d]
{{d → 0.00323984}, {d → 0.0541213}}
```

The IOL now sits 3.24 mm behind the cornea, so it had to move 2.34 mm

Question 3

PMMA has an index of n = 1.49. The thickness t = 1 mm. Equiconvex means that the front and back radii R1 = -R2. The power of each surface is given by

```
n = 1.49
ϕ1 = (n - 1.336) / R1
ϕ2 = -(1.336 - n) / R1
```

```
1.49
```

$$\frac{0.154}{R1}$$

$$\frac{0.154}{R1}$$

Next solve the thick lens power equation for R1

```
t = 0.001
ϕ = 20
Solve[ϕ == ϕ1 + ϕ2 - t * ϕ1 * ϕ2 / n, R1]
```

```
0.001

20

{{R1 → 0.0000518524}, {R1 → 0.0153481}}
```

So R1=15.35 mm. In air, the surface powers would be

```
ϕ1 = (n - 1) / 0.015348147558319293`
ϕ2 = -(1 - n) / 0.015348147558319293`
```

```
31.9257

31.9257
```

and the total power (D) would be

```
ϕ = ϕ1 + ϕ2 - t * ϕ1 * ϕ2 / n
```

```
63.1673
```

Question 4

Question 4

This problem seeks to create a +4.00 D spectacle lens that minimizes astigmatism. The basic setup is described in the problem. I added two features to the layout to facilitate the calculation. First, I made radius of the posterior spectacle lens surface a variable and second I added the merit function operand shown below, which drives the height of the on-axis marginal ray to zero at the image plane.

Merit Function Editor: 6.167733E-012

Edit Tools View Help

Oper #	Type	Surf	Wave	Hx	Hy	Px	Py
1 REAY	REAY	4	1	0.000000	0.000000	0.000000	1.000000

I then wrote a little macro to export the astigmatism values:

```
for R1=10,100,1
   radi(1)=R1
   optimize
   update all
   code=ocod("ASTI")
   astig=opev(code,0,1,0,1,0,0)
   print R1,",",astig
next
```

The for loop sets the radius of the anterior spectacle lens surface to R1 and then optimizes the system based on the merit function above. The optimization effectively sets the posterior radius to a value that keeps the lens power constant. The ocod and opev lines in the macro extract the Seidel astigmatism from Zemax and finally, the results are printed. A plot of the results is shown below. The approximate solutions for R1 are 29 mm and 48.5 mm.